# Trade Diversion beyond IIA*

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**Abstract**

In canonical CES trade models, a supplier's lost share is reallocated across rivals in proportion to their benchmark shares. Within a globally regular class of homothetic demand systems, we relax this restriction and provide a one-to-one map from positive diversion allocations to curvature vectors. We show that this flexibility changes rival shares at first order while preserving the CES cost-of-living response through second order. In public USITC data on U.S. textile and apparel imports during the U.S.–China trade war, Vietnam's 2018–2019 share gain is 1.14 percentage points, while CES assigns 0.52 points conditional on China's observed decline. We show that a flexible demand system calibrated conditional on China's observed decline reproduces the larger gain and quantifies the corresponding reallocation across other suppliers.

*Keywords*: Trade diversion; independence of irrelevant alternatives; trade elasticities
*JEL Classification*: D11; F11; F13; F14

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# 1 Introduction

When one preeminent exporter becomes more expensive, which of its competitors gains its lost market share? Under canonical CES, each rival gains in proportion to its initial share among the unaffected suppliers, regardless of the trade elasticity. Fréchet productivity draws in the Ricardina model of Eaton and Kortum (2002) imply the same restriction. These specifications impose independence of irrelevant alternatives (IIA): changing one supplier's source-specific price leaves the relative shares of two unaffected suppliers unchanged.

We show that benchmark shares and the affected exporter's local trade elasticity do not determine how its lost market share is divided among competitors within a globally regular homothetic demand family. For an affected exporter $k$, let $\theta > 0$ denote its local *own-odds* elasticity and $s$ the initial shares. Define $q_i \equiv s_i/(1 - s_k)$ as competitor $i$'s initial share among the unaffected suppliers. We construct a flexible but parsimonious demand system for which local diversion satisfies[1]

$$\mathcal{D}^{\text{CES}}_{i\leftarrow k} = q_i, \qquad \mathcal{D}_{i\leftarrow k} = \underbrace{q_i}_{\text{CES allocation}} \times \underbrace{r_i}_{\text{diversion multiplier}}, \qquad r_i \equiv (\sigma_i - 1)/\theta. \tag{1}$$

The diversion multiplier $r_i$ measures how much more or less of the affected exporter's lost market share competitor $i$ receives relative to CES. When $r_i = 1$ for all $i \neq k$, competitor $i$ gains exactly what CES predicts, while $r_i > 1$ ($r_i < 1$) means that it gains more (less) than CES predicts.[2] For intuition, think of $k$ as China and $i$ as Vietnam. CES assigns Vietnam a share of China's lost market share equal to Vietnam's initial share among all non-China suppliers, while we allow Vietnam to gain more or less through its diversion multiplier $r_i$.

We show that the construction delivers every positive diversion allocation while preserving global regularity. With several independently identified source-price shocks, its cross-shock restrictions can then be tested. For a given pair of rivals, their relative response, divided by the affected source's share, must be the same across all other sources.

Our results show that the composite price index under our demand system agrees with CES through second order around the benchmark, even though rival shares can differ at first order. Figure 1 shows an example with three equally sized suppliers. CES divides the affected supplier's lost market share equally between the two rivals, while our demand system assigns three-quarters to one rival with the same trade elasticity for the affected supplier. At a 25 percent price increase, relative spending on the two rivals changes by 24.46 percent under

[1] Holding nominal expenditure and all rival prices fixed, we define $\mathcal{D}_{i\leftarrow k} \equiv (\partial s_i/\partial \log p_k)/(-\partial s_k/\partial \log p_k)$ as the fraction of exporter $k$'s marginal share loss absorbed by competitor $i$.

[2] Given any strictly positive target diversion vector $d$ summing to one, setting $r_i = d_i/q_i$, or equivalently $\sigma_i = 1 + \theta r_i$, delivers $\mathcal{D}_{i\leftarrow k} = d_i$ for every competitor $i$ while preserving the initial observed souring shares and the affected exporter's local trade elasticity.

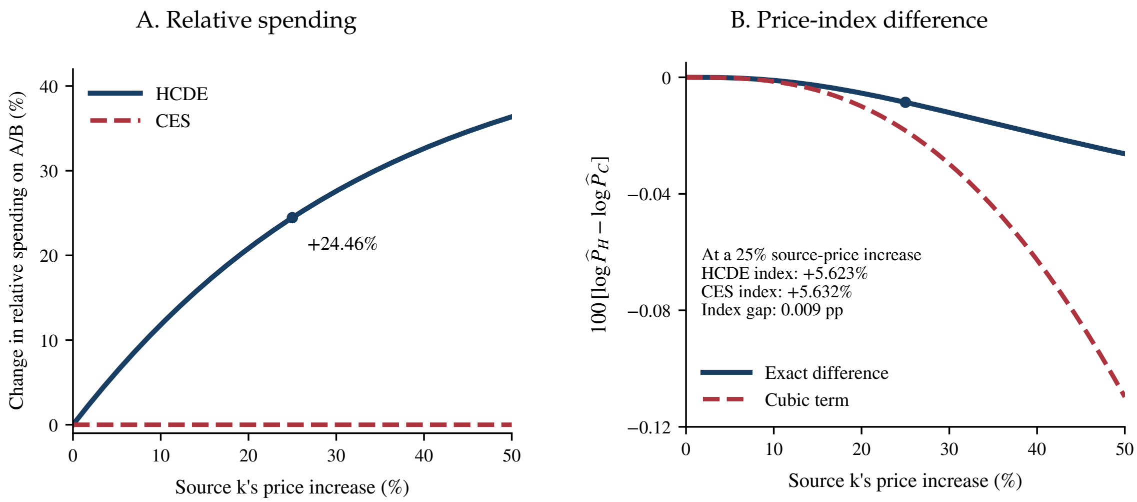


Figure 1: Different Diversion with Similar Aggregate Price Responses

*Notes:* The three suppliers have equal benchmark shares. CES uses $\sigma = 5$ and HCDE uses $(\sigma_A, \sigma_B, \sigma_k) = (7, 3, 5)$; only $p_k$ changes. Panel A compares the change in relative spending on the two unaffected suppliers, $A/B$, under HCDE and CES. Panel B compares the exact HCDE–CES composite price-index difference , $100(\log \widehat{P}_H - \log \widehat{P}_C)$, with the cubic approximation in Corollary 1. The markers report the 25% source-price increase.

our demand system and stays fixed under CES, yet the two models deliver nearly identical changes in the composite price index.

We illustrate the allocation margin with public U.S. textile and apparel imports. Between 2018 and 2019, China's share falls by 3.23 percentage points and Vietnam's rises by 1.14 points. Conditional on China's decline, CES assigns 0.52 points to Vietnam regardless of the trade elasticity. Vietnam's observed gain is 2.18 times this allocation. We calibrate our demand system to Vietnam's and Bangladesh's gains, then derive the full range of finite fits to every supplier's share. The fitted allocation does not depend on the trade elasticity.

Our demand system is the homothetic constant-difference-of-elasticities (HCDE) specification of the implicit-additivity class introduced by Hanoch (1975). Yang (2026*b*) introduces HCDE sourcing into a sectoral trade model, while Yang (2026*a*) shows how this class of implicit demand systems admits an exact-hat representation and separates exact-hat solvability from reference-share welfare compression. Kortum and Yang (2026) provide a guide to exact-hat counterfactual analysis in nested trade models with embedded implicit demand.

Piveteau and Smagghue (2024) develop mixed-CES sourcing, estimate departures from IIA, and study exporter-specific trade reallocation. Lind and Ramondo (2023) allow correlated productivity and derive non-CES Ricardian sourcing. Adão, Costinot and Donaldson (2017) characterize the demand information required for counterfactual predictions more generally. We complement these approaches with a parsimonious, globally regular demand system that can generate any positive diversion allocation while holding the affected source's local trade elasticity fixed. Our cost-of-living result also complements Arkolakis, Costinot and Rodríguez-Clare (2012) by showing that similar aggregate price responses can conceal different reallocations across suppliers.

## 2 The Sourcing Restriction

Fix a destination with $K \geq 3$ source countries, source-specific prices $p_i > 0$, nominal expenditure $m > 0$, and interior expenditure shares $s_i = p_i x_i / m$. For distinct sources $i, j, k$, define

$$\mathcal{I}^C_{ij,k} = \left.\frac{\partial \log(s_i/s_j)}{\partial \log p_k}\right|_u, \qquad \mathcal{I}^M_{ij,k} = \left.\frac{\partial \log(s_i/s_j)}{\partial \log p_k}\right|_m. \tag{2}$$

The compensated derivative $\mathcal{I}^C_{ij,k}$ holds utility fixed and allows expenditure to adjust, while the fixed-spending derivative $\mathcal{I}^M_{ij,k}$ holds expenditure fixed and allows utility to adjust.[3] We call the all-triples restrictions $\mathcal{I}^C_{ij,k} = 0$ and $\mathcal{I}^M_{ij,k} = 0$ compensated and fixed-spending price-IIA, respectively. They require a change in source $k$'s price to leave the relative expenditure shares of the two unaffected sources $i$ and $j$ unchanged. Since $p_i$ and $p_j$ are fixed, the same restrictions apply to their quantity ratio. We consider price changes within an existing sourcing set. With only two sources, the third-price restriction is vacuous.

Under homothetic CES,

$$s_i = \frac{a_i p_i^{1-\sigma}}{\sum_\ell a_\ell p_\ell^{1-\sigma}}, \qquad \frac{s_i}{s_j} = \frac{a_i}{a_j}\left(\frac{p_i}{p_j}\right)^{1-\sigma}. \tag{3}$$

Both derivatives in (2) vanish. We use this ratio restriction to obtain the finite allocation directly. For $s_k' \neq s_k$, define

$$\mathcal{D}^F_{i\leftarrow k} = \frac{s_i' - s_i}{s_k - s_k'}. \tag{4}$$

**Lemma 1 (Exact proportional diversion)** *Under CES, a change in $p_k$ alone implies $\mathcal{D}^F_{i\leftarrow k} = q_i$ for every rival, whenever the denominator is nonzero. The identity does not depend on the elasticity or the size of the price change.*

**Proof.** Write $Z = 1 - s_k + s_k \widehat{p}_k^{1-\sigma}$. Every rival has $s_i' = s_i/Z$, and adding up gives $s_k - s_k' = (1 - s_k)(1/Z - 1)$. Dividing the rival's gain by this loss gives $q_i$. $\square$

[3] The same share restrictions apply to analogous allocation problems in consumption and production, with output replacing utility in the production case.

The same argument applies to any fixed-spending share system whose single-source change leaves all rival ratios unchanged. At a zero share change the finite fraction is undefined, although the local limit is $q_i$.

In Eaton and Kortum (2002), independent productivity draws $F_i(z) = \exp(-T_i z^{-\theta})$ with common shape $\theta$ imply destination $n$'s sourcing shares

$$\pi_{ni} = \frac{T_i(c_i\tau_{ni})^{-\theta}}{\sum_\ell T_\ell(c_\ell\tau_{n\ell})^{-\theta}}, \qquad \frac{\pi_{ni}}{\pi_{nj}} = \frac{T_i}{T_j}\left(\frac{c_i\tau_{ni}}{c_j\tau_{nj}}\right)^{-\theta}. \tag{5}$$

Holding the unit and bilateral costs of $i, j$ fixed, a third source's cost leaves this ratio unchanged. The sourcing correspondence with CES uses $\theta = \sigma - 1$. Armington differentiation permits demand other than CES, and Fréchet marginal distributions permit correlation (Lind and Ramondo, 2023). In equilibrium, a shock to $k$ can change the wages and unit costs of $i, j$, so their total relative response can differ from the conditional derivative in (5).

Our framework can also allow CES elasticities to differ across sectors, as in Caliendo and Parro (2015), or nest suppliers within groups. A nested CES model allows non-proportional diversion across groups, but requires a partition of suppliers and preserves IIA within each common CES nest. In particular, placing CDE over sectoral composites does not remove IIA from a CES sourcing nest below them. Aggregation across sectors or nests can therefore change aggregate source ratios even when each conditional sourcing equation satisfies IIA. We apply the restriction at the level at which sources compete.

## 3 A Regular Demand System beyond IIA

We begin with the CDE expenditure representation

$$\sum_{i=1}^{K} \beta_i u^{e_i(1-\sigma_i)} \left(\frac{p_i}{C(p,u)}\right)^{1-\sigma_i} = 1, \qquad \beta_i > 0, \quad e_i > 0, \quad \sigma_i > 1. \tag{6}$$

The $e_i$ are expansion parameters and the $\sigma_i$ are partial-substitution parameters (Hanoch, 1975). Common $\sigma_i$ gives non-homothetic CES (NHCES), as used in structural-change models (Comin, Lashkari and Mestieri, 2021; Sposi, Yi and Zhang, 2026). Common $e_i$ gives homothetic CDE. We normalize the common expansion parameter to one through the utility index. [4] The restriction $\sigma_i > 1$ permits the trade construction below and avoids the separate limiting representation at $\sigma_i = 1$.

[4]Canonical CES sourcing uses the same normalization with common $\sigma_i = \sigma$.

Let $t_i = \beta_i u^{e_i(1-\sigma_i)}(p_i/C)^{1-\sigma_i}$. Implicit differentiation gives

$$s_i = \frac{(\sigma_i - 1)t_i}{\sum_\ell(\sigma_\ell - 1)t_\ell}, \qquad \bar{\sigma} = \sum_i s_i\sigma_i, \qquad \bar{e} = \sum_i s_i e_i. \tag{7}$$

The substitution parameters differ from own-price and pairwise elasticities. For $i \neq k$, the Allen–Uzawa elasticity is $\sigma_i + \sigma_k - \bar{\sigma}$ (Surry, 1993; Chen, 2017).

**Proposition 1 (Regularity)** *Equation (6) defines a unique positive expenditure function at every positive $(p, u)$. It is smooth, increasing in prices and utility, homogeneous of degree one in prices, and concave in prices.*

**Proof.** The left-hand side increases strictly from zero to infinity as $C$ increases. Differentiation yields $C_{p_i} = Cs_i/p_i > 0$ and $C_u = C\bar{e}/u > 0$. Rescaling prices rescales $C$. For any direction $z$, let $v_i = z_i/p_i$ and $\mu = \sum_i s_i v_i$. The price Hessian satisfies

$$\frac{z^{\mathsf{T}}C_{pp}z}{C} = -\sum_i s_i\sigma_i(v_i - \mu)^2 \leq 0. \tag{8}$$

Appendix A.1 derives the identity. □

**Proposition 2 (Two forms of IIA)** *For distinct $i, j, k$, CDE implies*

$$\mathcal{I}^C_{ij,k} = s_k(\sigma_i - \sigma_j), \tag{9}$$

$$\mathcal{I}^M_{ij,k} = -\frac{s_k}{\bar{e}}\left[e_i(1-\sigma_i) - e_j(1-\sigma_j)\right]. \tag{10}$$

*Compensated price-IIA holds for all triples if and only if all $\sigma_i$ are equal. Fixed-spending price-IIA holds for all triples if and only if all $e_i(1-\sigma_i)$ are equal.*

**Proof.** The log share ratio is a constant plus $[e_i(1-\sigma_i) - e_j(1-\sigma_j)]\log u + (1-\sigma_i)\log p_i - (1-\sigma_j)\log p_j + (\sigma_i - \sigma_j)\log C$. At fixed utility, $\partial \log C/\partial \log p_k = s_k$. At fixed spending, $C = m$ and $\partial \log u/\partial \log p_k = -s_k/\bar{e}$. Substitution gives both derivatives. Positivity and $K \geq 3$ establish the equivalences. □

We separate substitution from income effects in Table 1. NHCES preserves compensated IIA while income effects can change fixed-spending ratios. HCDE removes these income effects but allows nonzero third-price responses through heterogeneous curvature. In unrestricted CDE, setting $e_i = 1/(\sigma_i - 1)$ restores fixed-spending IIA even with heterogeneous $\sigma_i$.

Table 1: Homotheticity and Price-Based IIA

| Specification | Homothetic | Compensated IIA | Fixed-spending IIA |
|---|---|---|---|
| CES | Yes | Yes | Yes |
| NHCES | Not generally | Yes | Common $e_i$ |
| HCDE | Yes | Common $\sigma_i$ | Common $\sigma_i$ |
| CDE | Not generally | Common $\sigma_i$ | Common $e_i(1-\sigma_i)$ |

*Notes:* The displayed parameter conditions are necessary and sufficient within the branch in (6), with at least three sources. NHCES imposes common $\sigma_i$ and HCDE imposes common $e_i$. Responses condition on all other source prices.

We set $e_i = 1$ for the diversion analysis. Then $C(p,u) = uP(p)$ and

$$\sum_i \beta_i \left(\frac{p_i}{P}\right)^{1-\sigma_i} = 1. \tag{11}$$

Following Yang (2026*a*), we derive the benchmark aggregator weights from expenditure shares:

$$H(s,\sigma) = \sum_i \frac{s_i}{\sigma_i - 1}, \qquad \omega_i = \frac{s_i/(\sigma_i - 1)}{H(s,\sigma)}. \tag{12}$$

Let $\widehat{X} = X'/X$, the exact-hat system is

$$\sum_i \omega_i \left(\frac{\widehat{p}_i}{\widehat{P}}\right)^{1-\sigma_i} = 1, \qquad s_i' = \frac{s_i(\widehat{p}_i/\widehat{P})^{1-\sigma_i}}{\sum_\ell s_\ell(\widehat{p}_\ell/\widehat{P})^{1-\sigma_\ell}}. \tag{13}$$

The index equation increases strictly in $\log \widehat{P}$ and has a unique solution between the smallest and largest price changes. For two unaffected suppliers,

$$\frac{s_i'/s_j'}{s_i/s_j} = (\widehat{P})^{\sigma_i - \sigma_j}. \tag{14}$$

Heterogeneous curvature allows the common index to change relative shares.

## 4 The Allocation of Trade Diversion

Holding spending and all prices except $p_k$ fixed, define

$$\varepsilon_k = -\frac{\partial \log[s_k/(1-s_k)]}{\partial \log p_k}, \qquad \mathcal{D}_{i\leftarrow k} = \frac{\partial s_i/\partial \log p_k}{-\partial s_k/\partial \log p_k}, \quad i \neq k. \tag{15}$$

We normalize by the own-odds elasticity because it equals the common cost coefficient $\theta$ in CES and canonical EK sourcing when rival costs and source characteristics are fixed. This removes the factor $1 - s_k$ from the own log-share elasticity. In a heterogeneous demand system, a gravity coefficient need not equal this conditional derivative without the corresponding identifying restrictions. Diversion in (15) allocates a share loss, not a quantity loss.

HCDE implies the share Jacobian

$$J_{ik} \equiv \frac{\partial \log s_i}{\partial \log p_k} = (1 - \sigma_i)\mathbf{1}\{i = k\} + s_k(\sigma_i + \sigma_k - \bar{\sigma} - 1). \tag{16}$$

Hence, for $q_i = s_i/(1 - s_k)$ and $\varepsilon_k > 0$,

$$\mathcal{D}_{i \leftarrow k} = q_i \frac{\sigma_i + \sigma_k - \bar{\sigma} - 1}{\varepsilon_k}. \tag{17}$$

The numerator can differ across rivals even when the denominator is fixed.

**Theorem 1 (Same trade elasticity, arbitrary positive diversion)** *Fix an interior benchmark $s$, a source $k$, and $\theta > 0$. Every vector $(d_i)_{i \neq k}$ with $d_i > 0$ and $\sum_{i \neq k} d_i = 1$ is attained by a globally regular HCDE system reproducing $s$ and satisfying $\varepsilon_k = \theta$. The construction is*

$$\sigma_k = 1 + \theta, \qquad \sigma_i = 1 + \theta \frac{d_i}{q_i} \quad (i \neq k). \tag{18}$$

*Within the family normalized by $\sigma_k = 1 + \theta$ and satisfying $\varepsilon_k = \theta$, this is a bijection between curvature vectors with $\sigma_i > 1$ and the positive diversion simplex. Local diversion has $K - 2$ free dimensions.*

**Proof.** The constructed parameters exceed one and satisfy $\bar{\sigma} = 1 + \theta = \sigma_k$. Normalize benchmark prices and the index to one and set $\beta_i = \omega_i$ from (12). This reproduces $s$, and Proposition 1 establishes global regularity. Equation (16) gives $J_{kk} = -\theta(1 - s_k)$ and $\partial s_i/\partial \log p_k = s_i s_k(\sigma_i - 1)$ for $i \neq k$. Thus $\varepsilon_k = \theta$ and $\mathcal{D}_{i \leftarrow k} = d_i$. Conversely, imposing $\sigma_k = 1 + \theta$ and $J_{kk} = -\theta(1 - s_k)$ in (16) gives $\bar{\sigma} = \sigma_k$. Equation (17) then uniquely gives (18). Positivity and adding up follow. □

We choose the fixed demand weights separately for each curvature vector to reproduce the same initial shares, then hold those weights constant during the price change. The normalization selects an explicit family of HCDE systems consistent with the own-response moment. It leaves the full matrix of responses to other source-price changes restricted.

Let $x = \log \widehat{p}_k$, and write $f_H(x)$ and $f_C(x)$ for the log index changes under the constructed HCDE and its CES benchmark. Define $V = \sum_i s_i[\sigma_i - (1 + \theta)]^2$.

**Corollary 1 (Index comparison)** *Under Theorem 1, $f_H'(0) = f_C'(0) = s_k$ and $f_H''(0) = f_C''(0) =$*

$-\theta s_k(1-s_k)$. *Moreover,*

$$f_H(x) - f_C(x) = -\frac{s_k^3 V}{6} x^3 + O(x^4). \tag{19}$$

*Non-proportional diversion changes some rival shares at first order, while the log indices agree through second order.*

**Proof.** Shephard's lemma gives $f'(x) = s_k(x)$. Matching $s_k$ and $J_{kk}$ therefore matches the first two derivatives. At the constructed benchmark, $\mathrm{d}\bar{\sigma}/\mathrm{d}x = s_k V$. Differentiating (16) gives $s''_{k,H}(0) - s''_{k,C}(0) = -s_k^3 V$, which yields (19). Appendix A.2 provides the intermediate steps. □

For the 25 percent shock in Figure 1, the exact log-index gap is $-8.59 \times 10^{-5}$ and the cubic term is $-1.83 \times 10^{-4}$. Higher-order terms offset about 53 percent of the cubic term. We use the exact index equation for finite changes. The corollary establishes the local distinction between the orders at which diversion and aggregate cost differ.

### 4.1 Restrictions across Shocks

For a fixed rival pair, CDE implies

$$\frac{\mathcal{I}^C_{ij,k}}{s_k} = \sigma_i - \sigma_j, \qquad (\sigma_i - \sigma_j) + (\sigma_j - \sigma_\ell) = \sigma_i - \sigma_\ell. \tag{20}$$

The normalized relative response is independent of which excluded source changes price. On a rectangular set of rival pairs and price shifters excluded from every pair, the unnormalized response matrix has rank at most one. The full own-and-cross-price matrix is not subject to that rank bound. With at least four sources, independently identified changes in two excluded prices can test the equality for a given rival pair. These restrictions hold for compensated CDE and fixed-spending HCDE. They can be tested in a second, independently identified source-price episode after fitting the first episode. Observing additional supplier shares within the first episode does not supply this cross-shock variation.

Global regularity permits compensated complementarity between some rivals. For four equal shares, $\theta = 4$, and $d = (0.9, 0.05, 0.05)$ away from source four, (18) gives $\sigma = (11.8, 1.6, 1.6, 5)$. The Allen–Uzawa elasticity between rivals two and three is $-1.8$, even though every rival locally gains from an increase in source four's price. Requiring every pair to be compensated substitutes imposes $\sigma_i + \sigma_j \geq \bar{\sigma}$ and reduces the set of possible diversion patterns. Richer pair-specific patterns can be studied using nests, characteristics, or correlated productivity (Piveteau and Smagghue, 2024; Lind and Ramondo, 2023).

### 4.2 Allowing Negative Diversion

We can preserve the affected source's elasticity while allowing some rivals to lose spending. Write $a_i = \sigma_i - 1$, $s = s_k$, and $t = \bar{\sigma} - \sigma_k$. For a signed local diversion vector with $\sum_{i \neq k} d_i = 1$, equation (17) and the own-response condition give

$$a_k = \theta - \frac{s}{1-s}t, \qquad a_i = \theta \frac{d_i}{q_i} + t \quad (i \neq k). \tag{21}$$

All curvatures exceed one if and only if

$$-\theta \min_{i \neq k} \frac{d_i}{q_i} < t < \theta \frac{1-s}{s}. \tag{22}$$

To verify these statements, substitute $a_i = \theta d_i / q_i + t$ into $\bar{\sigma} - \sigma_k = t$ and solve for $a_k$. Positivity of the resulting $a_i$ gives both bounds. The interval is nonempty exactly when $d_i > -s_i/s$ for every rival. Thus a negative local allocation can be feasible under global regularity, while the normalization $t = 0$ in Theorem 1 restricts local diversion to be positive.

A negative fixed-spending response need not imply compensated complementarity. The Allen–Uzawa elasticity with source $k$ is $1 + \theta d_i / q_i$, which is negative only if $d_i / q_i < -1/\theta$. The distinction between local and finite responses also matters. Since $\bar{\sigma}$ changes along a price path, a rival with positive initial diversion can eventually lose share.[5] Matching the benchmark share and elasticity continues to imply second-order index agreement in this larger family. The specific cubic coefficient in Corollary 1 uses $t = 0$.

## 5 Trade Reallocation in Public Import Data

We focus on U.S. textile and apparel imports from 2018 to 2019, during the U.S.–China trade-war period, when China's sourcing share fell and alternative Asian suppliers gained market share. This episode shows how lost sourcing from a large exporter is reallocated across competitors. We use U.S. general imports from United States International Trade Commission (2023), Table 2, in a common data vintage. We keep ten suppliers separately identified and group all other origins into a residual category. Our calibration measures the curvature heterogeneity needed to rationalize the observed allocation under a single-source-price change, with supply-side reallocation providing an observationally equivalent explanation at this level of aggregation. Total imports rise from $126,992$ million to $127,238$ million. China's share falls from 36.864 to 33.631 percent and Vietnam's rises from 10.178 to 11.314 percent.

[5] With equal shares, $\sigma = (8.92, 1.08, 5)$ gives local diversion $(0.99, 0.01)$ away from source three. A 25 percent rise in its price reduces source two's share by 0.435 percentage points. The positivity in Theorem 1 therefore does not impose a global sign restriction on finite share changes.

Table 2: Observed Reallocation and the Exact CES Allocation, 2018–2019

| | Import share (%) | | Change (pp) | | Relative |
|---|---|---|---|---|---|
| Source | 2018 | 2019 | Observed | CES | allocation |
| China | 36.864 | 33.631 | -3.232 | -3.232 | — |
| Vietnam | 10.178 | 11.314 | +1.136 | +0.521 | 2.181 |
| India | 6.719 | 6.991 | +0.272 | +0.344 | 0.792 |
| Bangladesh | 4.475 | 4.859 | +0.384 | +0.229 | 1.674 |
| Mexico | 4.702 | 4.611 | -0.091 | +0.241 | -0.377 |
| Indonesia | 3.878 | 3.838 | -0.040 | +0.199 | -0.200 |
| Pakistan | 2.410 | 2.563 | +0.153 | +0.123 | 1.236 |
| Cambodia | 1.973 | 2.225 | +0.252 | +0.101 | 2.499 |
| Honduras | 2.113 | 2.300 | +0.187 | +0.108 | 1.728 |
| Italy | 1.911 | 1.946 | +0.035 | +0.098 | 0.356 |
| Other origins | 24.778 | 25.722 | +0.944 | +1.269 | 0.744 |

*Notes:* The CES column holds China's observed share decline fixed and reallocates it across rivals in proportion to their 2018 shares among non-China suppliers, $q_i$. The superscript $F$ denotes finite, rather than local, diversion. Relative allocation, $\mathcal{D}^F_{i\leftarrow C}/q_i$, compares the observed finite diversion with the CES benchmark: one denotes proportional diversion, values above (below) one indicate more (less) reallocation than CES, and negative values indicate that the rival's share also fell. Source: United States International Trade Commission (2023).

The pattern relates to the third-country reallocation studied by Fajgelbaum et al. (2024).

## 5.1 An Exact CES Benchmark

Vietnam's gain divided by China's loss is 35.16 percent, compared with an initial share of 16.12 percent among non-Chinese suppliers. Lemma 1 assigns the latter fraction of China's loss to Vietnam under CES, whatever the elasticity. The implied gain is 0.52 percentage points, compared with 1.14 points in the data. Table 2 reports this comparison for each supplier.

The exposure-adjusted allocation $\mathcal{D}^F_{i\leftarrow C}/q_i$ is 2.18 for Vietnam and 2.50 for Cambodia. Mexico and Indonesia have negative entries because their shares fall alongside China's. These finite accounting ratios need not equal local diversion. Theorem 1 describes the latter and requires positive entries.

## 5.2 Conditional Diversion Multipliers

We first use a parsimonious calibration to connect the normalized theorem to Vietnam's gain. Set $\theta = 4$ and write

$$d_i = \frac{q_i r_i}{\sum_{\ell \neq C} q_\ell r_\ell}, \qquad r_i > 0. \tag{23}$$

With all other diversion multipliers set to one, we choose Vietnam's diversion multiplier $r_V$ and the effective Chinese log price change $x$ to satisfy

$$s'_C(r_V, x) = s_C^{2019}, \qquad s'_V(r_V, x) = s_V^{2019}, \qquad \widehat{p}_C = \exp(x). \tag{24}$$

Solving the exact-hat system gives $r_V = 2.72$ and a 3.60 percent price increase. Adding Bangladesh's observed share as a third moment gives $r_V = 3.04$, $r_B = 2.36$, and a 3.60 percent price increase. The curvatures are $(\sigma_C, \sigma_V, \sigma_B, \sigma_R) = (5, 9.52, 7.63, 3.81)$.

At the common Chinese price increase of 3.5959 percent, CES and the two-tilt HCDE indices rise by 1.2528 and 1.2527 percent, a gap of 0.0105 basis points, while Vietnam receives 2.18 times the CES fraction of displaced spending. This comparison illustrates the allocation and index results together.

The remaining rivals have common curvature $\sigma_R$ and unchanged prices, so their relative shares remain fixed. Once China, Vietnam, and Bangladesh are fitted, adding up determines this group's total gain and its initial shares determine the allocation within it. We use all supplier movements in the finite-fit analysis below.

Figure 2 varies Vietnam's diversion multiplier with Bangladesh's held at its calibrated value, fitting China's decline at each point. The marker also reproduces Vietnam's gain. Finite diversion differs slightly from local diversion as shares change along the price path.

### 5.3 The Range of Finite-Share Fits

We next fit every supplier's observed share change. Let $g_i = \log(s'_i/s_i)$, $z = \log \widehat{P}$, $x = \log \widehat{p}_C$, and $\delta$ be the log change in the share denominator.[6] The exact-hat equations imply

$$a_i = \frac{g_i + \delta}{z}\ (i \neq C), \qquad a_C = -\frac{g_C + \delta}{x - z}, \qquad e^{\delta} \sum_i \frac{s'_i}{a_i} = \sum_i \frac{s_i}{a_i}. \tag{25}$$

A regular finite fit exists exactly when $g_C < g_i$ for every rival, as in these data. Fixing $\varepsilon_C = 4$ sets the scale, while one scalar remains to select a curvature vector. Appendix A.3 gives the full family, indexed by

$$\delta_L \equiv \max_{i \neq C}(-g_i) < \delta < \delta_U \equiv -g_C.$$

Here $(\delta_L, \delta_U) = (0.019506, 0.091766)$, corresponding to $1.480 < t < 6.851$ for $t = \bar{\sigma} - \sigma_C$. These are the bounds for exact finite fits, distinct from the local bounds in (22).

Table 3 reports the boundary values of this family. Every interior member reproduces all

[6]Here $\delta$ indexes the remaining one-dimensional freedom among exact fits. Varying $\delta$ changes the fitted curvatures jointly while leaving the observed finite-share reallocation unchanged; across rival suppliers, the ordering and relative pattern of curvature differences are preserved up to a common factor. Its admissible range requires $\sigma_i > 1$ for all suppliers.

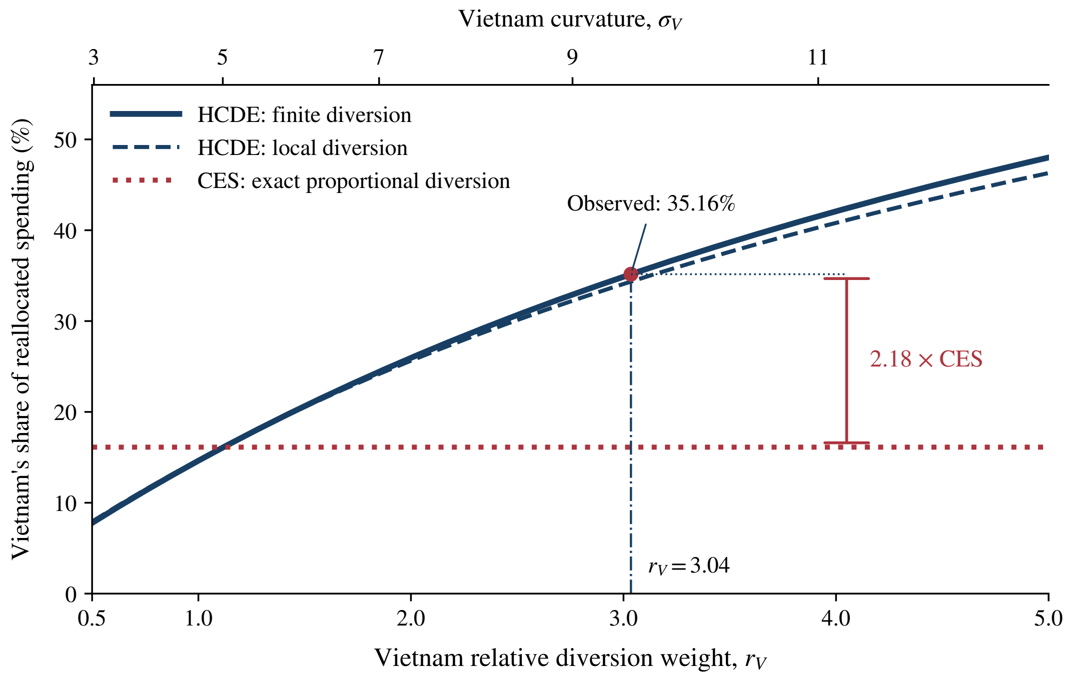


Figure 2: Diversion Conditional on the Observed Chinese Share Decline

*Notes:* The 2018 sourcing shares and $\theta = 4$ are fixed throughout, with $r_B = 2.36$ and all rival diversion multipliers other than Vietnam's and Bangladesh's set to one. The solid blue line shows *HCDE finite diversion* to Vietnam as $r_V$ varies, with the Chinese price change re-solved at each point to match China's observed 3.23-pp share decline. The dashed blue line shows the corresponding *HCDE local diversion* at the 2018 benchmark. The red dotted line shows the *CES prediction: exact proportional diversion*, which is fixed at 16.12%. The marker at $r_V = 3.04$ matches Vietnam's observed finite diversion of 35.16%, and the upper axis reports the corresponding $\sigma_V$.

eleven final shares. Vietnam's curvature varies from 11.000 to 16.212, but its Allen–Uzawa elasticity with China varies only from 9.361 to 9.519. The distinction also matters for the two rivals that lose share. Mexico's elasticity lies between −0.480 and −0.288, while Indonesia's lies between 0.255 and 0.421. At an elasticity of four, Mexico is a compensated complement to China across all possible fits. Indonesia loses share even though it remains a compensated substitute for China across all possible fits. The bounds follow from the index and elasticity expressions in Appendix A.3. An independently identified second source-price change can test the cross-shock restrictions in Section 4.1.

Table 3: Curvature and Substitution over the Finite-Fit Family

| Source | Curvature $\sigma_i$ | | Allen–Uzawa elasticity $S_{iC}$ | |
|---|---|---|---|---|
| | $\delta \downarrow \delta_L$ | $\delta \uparrow \delta_U$ | $\delta \downarrow \delta_L$ | $\delta \uparrow \delta_U$ |
| China | 4.136 | 1.000 | — | — |
| Vietnam | 11.000 | 16.212 | 9.519 | 9.361 |
| India | 5.725 | 11.122 | 4.245 | 4.271 |
| Bangladesh | 9.115 | 14.393 | 7.635 | 7.543 |
| Mexico | 1.000 | 6.562 | −0.480 | −0.288 |
| Indonesia | 1.735 | 7.271 | 0.255 | 0.421 |
| Pakistan | 7.450 | 12.787 | 5.970 | 5.936 |
| Cambodia | 12.161 | 17.332 | 10.680 | 10.482 |
| Honduras | 9.318 | 14.589 | 7.837 | 7.738 |
| Italy | 3.996 | 9.454 | 2.516 | 2.603 |
| Other origins | 5.538 | 10.942 | 4.058 | 4.091 |
| China price change (%) | 3.6221 | 3.7580 | — | — |
| Composite index change (%) | 1.2615 | 1.3076 | — | — |

*Notes:* Boundary limits of the exact fits to all 2019 shares, with $\theta = 4$. Both endpoints are excluded from $\sigma_i > 1$. The Allen–Uzawa elasticities are evaluated at the 2018 benchmark. Each column corresponds to one boundary, not to independently chosen supplier parameters.

### 5.4 Trade Elasticity and Interpretation

We show that the fitted allocation does not depend on the chosen trade elasticity. For any $c > 0$, the transformation

$$a_i \mapsto ca_i, \qquad \log \widehat{p}_i \mapsto c^{-1} \log \widehat{p}_i, \qquad \log \widehat{P} \mapsto c^{-1} \log \widehat{P} \tag{26}$$

preserves the weights and exponents in (13), hence every share, while multiplying $\varepsilon_k$ by $c$. The calibrated $r_V$ and $r_B$ are therefore unchanged for every $\theta > 0$. The full finite fit has the same invariance with $t/\theta$ fixed. Excess curvatures scale with $\theta$ and the implied log price change scales inversely. An independently observed price change would constrain this scaling.

## 6 Welfare and the Allocation Margin

Our comparison shows that substantially different reallocations across exporters can coexist with very similar changes in the cost of the import composite. The reference-share identity of Yang (2026*a*) connects this allocation margin to welfare accounting: welfare depends on a reference source's share change and a composition term that reflects expenditure shares across sources with different curvatures. Under common curvature, this composition term is constant across allocations, yielding the CES reduction to a single reference share. Our index corollary is conditional on a source-price change, the benchmark shares, and the affected source's local trade elasticity at the benchmark. Extending the comparison to national welfare additionally requires production and income restrictions, as in the sufficient-statistic result of Arkolakis, Costinot and Rodríguez-Clare (2012), and the corresponding equilibrium equations, as in Caliendo and Parro (2015).

## 7 Conclusion

We show that matching an exporter's initial market share and local trade elasticity places little restriction on where trade is diverted when its price changes. A parsimonious, globally regular demand system can accommodate any positive pattern of diversion while generating a log price index that agrees with CES through second order. Thus, substantially different reallocations across suppliers can have similar local effects on the aggregate cost of living. CES, in contrast, imposes proportional diversion at every shock size. Using historical trade data, we quantify these differences and show how source-price variation can test the remaining restrictions. More broadly, by allowing trade shocks to induce non-proportional reallocation across suppliers, the framework can help quantify their effects on global sourcing, third-country spillovers, welfare, and exposure to subsequent shocks.

# A Additional Derivations

## A.1 Compensated Derivatives and Concavity

Write $b_i = 1 - \sigma_i < 0$ and $t_i = \beta_i u^{e_i b_i}(p_i/C)^{b_i}$. Differentiating the defining equation gives

$$\mathrm{d}\log C = \sum_i s_i \mathrm{d}\log p_i + \bar{e}\mathrm{d}\log u, \qquad s_i = \frac{b_i t_i}{\sum_\ell b_\ell t_\ell}. \tag{A.1}$$

At fixed utility, $\mathrm{d}\log t_i = b_i(\mathrm{d}\log p_i - \mathrm{d}\log C)$. The common derivative term satisfies

$$\mathrm{d}\log\left|\sum_\ell b_\ell t_\ell\right| = \sum_\ell s_\ell \mathrm{d}\log t_\ell.$$

It follows that

$$\left.\frac{\partial \log s_i}{\partial \log p_k}\right|_u = b_i \mathbf{1}\{i = k\} + s_k(\sigma_i + \sigma_k - \bar{\sigma} - 1). \tag{A.2}$$

For Hicksian demand $h_i = Cs_i/p_i$, the elasticity is

$$\frac{\partial \log h_i}{\partial \log p_k} = -\sigma_i \mathbf{1}\{i = k\} + s_k(\sigma_i + \sigma_k - \bar{\sigma}). \tag{A.3}$$

The off-diagonal Allen–Uzawa elasticity is $CC_{ik}/(C_i C_k) = \sigma_i + \sigma_k - \bar{\sigma}$. For $v_i = z_i/p_i$ and $\mu = \sum_i s_i v_i$, the symmetric Hessian therefore satisfies

$$\begin{aligned}\frac{z^\mathsf{T} C_{pp} z}{C} &= -\sum_i s_i \sigma_i v_i^2 + 2\Big(\sum_i s_i \sigma_i v_i\Big)\mu - \bar{\sigma}\mu^2 \\ &= -\sum_i s_i \sigma_i (v_i - \mu)^2 \le 0. \end{aligned} \tag{A.4}$$

For HCDE, shares are independent of utility, so (A.2) also gives the fixed-spending Jacobian. For general CDE, fixed spending imposes $\mathrm{d}\log u = -\sum_i s_i \mathrm{d}\log p_i/\bar{e}$.

## A.2 Third-Order Index Comparison

Let $s = s_k(0)$, $\sigma_0 = 1 + \theta$, $z_i = \sigma_i - \sigma_0$, and $V = \sum_i s_i(0) z_i^2$. In the construction, $z_k = 0$ and $\sum_i s_i(0) z_i = 0$. Dots denote derivatives with respect to $x = \log \widehat{p}_k$, evaluated at zero. For rivals,

$$\dot{s}_i = s_i s(\theta + z_i), \qquad \dot{s}_k = -\theta s(1 - s).$$

Since the shares sum to one and curvatures are fixed,

$$\dot{\bar{\sigma}} = \sum_i \sigma_i \dot{s}_i = \sum_i z_i \dot{s}_i = s \sum_i s_i z_i(\theta + z_i) = sV. \tag{A.5}$$

Along the price path,

$$J_{kk} = 1 - \sigma_k + s_k(2\sigma_k - \bar{\sigma} - 1), \qquad \dot{J}_{kk} = \dot{s}_k(2\sigma_k - \bar{\sigma} - 1) - s_k \dot{\bar{\sigma}}.$$

The benchmark values of $s_k$, $\dot{s}_k$, and $2\sigma_k - \bar{\sigma} - 1 = \theta$ agree between models. CES has $\dot{\bar{\sigma}} = 0$, yielding $\dot{J}_{kk,H} - \dot{J}_{kk,C} = -s^2 V$. Because $f' = s_k$ and $\ddot{s}_k = s_k J_{kk}^2 + s_k \dot{J}_{kk}$,

$$f_H'''(0) - f_C'''(0) = -s^3 V.$$

Both log indices are zero initially and have the same first two derivatives. Smoothness and Taylor's theorem give (19). For the equal-share example, $z = (2, -2, 0)$ and $V = 8/3$, so the cubic coefficient is $-4/243$. The replication reports exact and cubic gaps at price increases of 1, 5, 10, 25, and 50 percent and checks the limiting coefficient at higher precision.

### A.3 Exact Finite-Share Inversion

Let $s$ and $s'$ be positive share vectors and set $g_i = \log(s_i'/s_i)$. For a price increase at source $k$ alone, define $x = \log \widehat{p}_k > 0$, $z = \log \widehat{P}$, and $a_i = \sigma_i - 1 > 0$. The hats imply $0 < z < x$ and

$$g_i = a_i z - \delta \quad (i \neq k), \qquad g_k = -a_k(x - z) - \delta,$$

where $e^\delta$ is the share denominator. Hence $g_i > g_k$ for every rival. This strict ordering is also sufficient for a regular finite fit with any initial own-odds elasticity $\theta > 0$.

To construct the fit, choose

$$\max_{i \neq k}(-g_i) < \delta < -g_k. \tag{A.6}$$

Write $v_i = g_i + \delta > 0$ for rivals and define

$$B(\delta) = \sum_{i \neq k} s_i \frac{e^{v_i} - 1}{v_i}, \qquad v_k = \frac{s_k - e^\delta s_k'}{B(\delta)} > 0, \qquad \bar{v} = \sum_i s_i v_i. \tag{A.7}$$

We then set

$$z = \frac{(1 - 2s_k)v_k + s_k \bar{v}}{(1 - s_k)\theta}, \qquad a_i = \frac{v_i}{z}, \qquad x = z\left(1 - \frac{g_k + \delta}{v_k}\right). \tag{A.8}$$

The numerator of $z$ equals $(1 - s_k)^2 v_k + s_k \sum_{i \neq k} s_i v_i > 0$. Thus $z > 0$, every $a_i > 0$, and $x > z$.

For the aggregator, the definition of $v_k$ gives

$$\sum_i \frac{e^\delta s_i' - s_i}{v_i} = 0.$$

Since $a_i = v_i/z$, this is the last equation in (25). The share equations hold by construction, so the price index and all final shares are reproduced. Equation (16) gives

$$\varepsilon_k = \frac{(1-2s_k)a_k + s_k \sum_i s_i a_i}{1-s_k} = \theta$$

by the choice of $z$. Proposition 1 establishes global regularity.

Every regular finite fit has a $\delta$ in (A.6) and takes this form. The offset satisfies

$$t = \frac{\bar{v} - v_k}{z}, \qquad \frac{t}{\theta} = \frac{(1-s_k)(y-1)}{1-2s_k+s_k y}, \qquad y = \frac{\bar{v}}{v_k}. \tag{A.9}$$

As $\delta$ increases, each rival's $v_i$ increases and $v_k$ decreases, so $y$ and $t$ increase. Thus an admissible offset selects a unique member.

We calculate Table 3 as limits at the two boundaries of (A.6). Let $\bar{g}_R = \sum_{i\neq k} s_i g_i/(1-s_k)$. Adding up and $\varepsilon_k = \theta$ give

$$S_{ik} = 1 + \theta + \frac{g_i - \bar{g}_R}{z}, \qquad i \neq k. \tag{A.10}$$

The index $z$ increases with $\delta$. To verify this, $B > 1 - s_k$ and $0 < B' < B$. Writing $r = e^{g_k+\delta} \in (0,1)$ gives

$$-v_k' = s_k\left(\frac{r}{B} + \frac{(1-r)B'}{B^2}\right) < \frac{s_k}{1-s_k}, \qquad z' = \frac{s_k + (1-s_k)v_k'}{\theta} > 0.$$

Equation (A.10) is therefore monotone for each rival and its boundary limits give the bounds. At the upper boundary, $v_k \to 0$ and $x \to z(1 + B/s_k)$. The lower boundary has at least one rival curvature tending to one. Both are excluded from the maintained branch.

## A.4 Calibration and Data Vintage

We set $\beta_i = \omega_i$ at unit benchmark prices and fit the selected share changes using (13). With common curvature and unchanged prices, rivals' contributions aggregate and their relative shares stay fixed. The replication provides solver checks and the axis transformation in Figure 2. A 2017–2019 check combines the 2017 benchmark from United States International Trade Commission (2018) with the 2019 endpoint from United States International Trade Commission (2023) and gives $r_V = 3.01$ in the one-tilt calibration.

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