Internal Trade Costs and the Optimal External Tariff

A three-region, two-country Eaton–Kortum derivation

Subject: How the optimal external tariff is derived from internal trade costs

0.1 Main result and economic interpretation

In the earlier two-region, two-country benchmark, the optimal tariff is

$$\tau^* = \frac{1}{\theta \pi^*_{mm}}, \tag{1}$$

where m denotes the foreign country and π_{mm} is its domestic expenditure share.

This note generalizes that benchmark to a three-region, two-country economy. Regions 1 and 2 belong to Home, while region 3 is the foreign country, or ROW. Home imposes a common external tariff τ on imports from region 3 into both Home regions.

The main result is

$$\boxed{\underbrace{\frac{\tau^*}{1+\tau^*}}_{\substack{\text{optimal common}\\ \text{tariff wedge}}} = \underbrace{\Psi^{R,*}_H}_{\substack{\text{regional-adjusted}\\ \text{Home market-power index}}} \underbrace{\frac{1}{1+\theta\pi^*_{33}}}_{\substack{\text{inverse ROW producer-value}\\ \text{export response}}}.} \tag{2}$$

All starred objects are evaluated at the optimal equilibrium. Equation (2) can be read as

$$\boxed{\text{optimal tariff wedge} = \frac{\text{regional-adjusted Home market-power force}}{\text{ROW producer-value export response}}.}$$

The formula separates the determination of the optimal tariff into a foreign response term,

$$1 + \theta\pi_{33},$$

and a Home-side regional term,

$$\Psi^R_H.$$

The first determines how difficult it is for Home to affect the price received by ROW producers. The second determines how Home's internal regional structure strengthens or weakens the gains from exercising that market power.

The optimal tariff wedge. The left-hand side of equation (2) is

$$\frac{\tau}{1+\tau}. \tag{3}$$

This is the policy wedge generated by the ad valorem tariff. It is also the fraction of tariff-inclusive import expenditure collected as tariff revenue. For Home region $h \in \{1, 2\}$, tariff revenue is

$$R_h = \frac{\tau}{1+\tau} X_h \pi_{h3}, \tag{4}$$

where X_h is total expenditure in region h, and π_{h3} is the share of that expenditure allocated to imports from ROW.

The wedge $\tau/(1+\tau)$, rather than the tariff rate τ itself, appears naturally in the optimality condition because bilateral expenditure $X_h \pi_{h3}$ is measured at the tariff-inclusive purchaser price, while ROW producers receive only the fraction $1/(1+\tau)$.

The ROW producer-value export response. The denominator of the optimal-tariff formula is

$$1 + \theta \pi_{33}. \tag{5}$$

Here, $\theta > 0$ is the Eaton–Kortum trade elasticity, and π_{33} is ROW's domestic expenditure share: the fraction of ROW expenditure allocated to goods produced in ROW itself.

To interpret this term, define ROW producer-value exports to Home as

$$E_{H3} \equiv \frac{1}{1+\tau} \sum_{h=1}^{2} X_h \pi_{h3}. \tag{6}$$

The factor $1/(1+\tau)$ converts Home's tariff-inclusive import expenditure into the value received by ROW producers.

ROW goods-market clearing implies

$$E_{H3} = w_3 L_3 (1 - \pi_{33}), \tag{7}$$

where $w_3 L_3$ is the value of ROW production.

The elasticity of ROW producer-value exports with respect to the ROW producer price is

$$\boxed{\varepsilon_3^V \equiv \frac{\partial \log E_{H3}}{\partial \log w_3} = 1 + \theta \pi_{33}.} \tag{8}$$

The term 1 is the mechanical value effect: holding export quantities fixed, a higher ROW producer price raises the value of exports.

The term $\theta \pi_{33}$ is the expenditure-share response. A higher ROW producer price reduces ROW consumers' expenditure share on ROW goods. This reallocation increases the share of ROW production available for sale to Home.

Thus, a larger $1 + \theta \pi_{33}$ means that ROW producer-value exports are more responsive to the ROW producer price. Home then has less ability to lower the price received by ROW producers, and the optimal tariff wedge is smaller, holding the Home-side term fixed.

The regional-adjusted Home market-power index. The Home-side term is

$$\boxed{\Psi_H^R \equiv \mathcal{S}_H \mathcal{I}_H.} \tag{9}$$

It consists of two distinct regional components:

$$\boxed{\Psi_H^R = \underbrace{\mathcal{S}_H}_{\substack{\text{conditional-sourcing} \\ \text{component}}} \underbrace{\mathcal{I}_H}_{\substack{\text{regional-incidence} \\ \text{component}}}.} \tag{10}$$

We refer to Ψ_H^R as the regional-adjusted Home market-power index because, after factoring out ROW's export response, it is exactly the remaining Home-side sufficient statistic determining the optimal tariff wedge.

It is not an additional behavioral assumption and is not a primitive elasticity. It is an endogenous object constructed from equilibrium trade shares, regional expenditures, and the shadow values implied by the government's first-order conditions.

The two components capture different ways in which the internal regional structure modifies Home's effective market power.

The conditional-sourcing component. The first component is

$$\boxed{\mathcal{S}_H \equiv \frac{\sum_{h=1}^{2} \omega_h^S \mu_h}{\mu_3}.} \tag{11}$$

This term compares two compositions of demand for goods produced in Home regions 1 and 2.

The numerator,

$$\sum_{h=1}^{2} \omega_h^S \mu_h,$$

is the shadow value of the production composition toward which the two Home regions substitute when the tariff raises the relative price of ROW goods.

The denominator,

$$\mu_3,$$

is the shadow value of the production composition that ROW purchases from the two Home regions.

Thus,

$$\mathcal{S}_H = \frac{\text{shadow value of tariff-induced Home sourcing}}{\text{shadow value of ROW sourcing from Home}}.$$

If $\mathcal{S}_H = 1$, tariff-induced substitution inside Home is directed toward the same shadow-value-weighted mixture of Home origins as ROW demand.

If $\mathcal{S}_H > 1$, the common tariff redirects Home expenditure toward a mixture of Home origins with a higher shadow value than the mixture purchased by ROW. This strengthens the Home-side tariff motive.

If $\mathcal{S}_H < 1$, tariff-induced substitution is directed toward a lower-shadow-value mixture of Home origins, weakening the tariff motive.

The regional-incidence component. The second component is

$$\boxed{\mathcal{I}_H \equiv \frac{\sum_{h=1}^{2} \omega_h^M \gamma_{xh}}{\sum_{h=1}^{2} \omega_h^S \gamma_{xh}}.} \tag{12}$$

The multiplier γ_{xh} is associated with region h's expenditure-income condition. Under the sign convention of the Lagrangian, it records the marginal shadow valuation of relaxing that regional constraint.

The numerator weights these regional shadow values by the geographical distribution of Home's imports from ROW.

The denominator weights the same shadow values by the geographical distribution of the tariff-induced change in import demand.

Hence,

$$\mathcal{I}_H = \frac{\text{shadow value where ROW imports and tariff revenue are located}}{\text{shadow value where the tariff changes import demand}}.$$

If $\mathcal{I}_H = 1$, the regional distribution of import exposure and tariff revenue is aligned, in shadow-value terms, with the regional distribution of the behavioral response to the tariff.

If $\mathcal{I}_H > 1$, imports and tariff revenue are concentrated in regions with relatively high shadow values compared with the regions in which the tariff induces the strongest substitution. This strengthens the effective Home-side policy force.

If $\mathcal{I}_H < 1$, the regional incidence of imports and revenue is less valuable than the regional pattern of tariff-induced substitution, weakening the policy force.

Conditional-sourcing shadow values. For each destination $n \in \{1, 2, 3\}$, define

$$\boxed{\mu_n \equiv \sum_{i=1}^{2} \gamma_i \frac{\pi_{ni}}{1 - \pi_{n3}}.} \tag{13}$$

The multiplier γ_i is associated with the market-clearing condition for goods produced in Home region i. It records the marginal shadow valuation of an additional unit of demand for that region's output under the chosen Lagrangian representation.

Because

$$\pi_{n1} + \pi_{n2} = 1 - \pi_{n3},$$

the ratio

$$\frac{\pi_{ni}}{1-\pi_{n3}} \tag{14}$$

is destination n's expenditure share on Home origin i, conditional on purchasing a Home-produced good.

Therefore, μ_n is the shadow-value-weighted average of destination n's conditional sourcing shares across the two Home origins.

For a Home destination $h \in \{1, 2\}$,

$$\mu_h = \gamma_1 \frac{\pi_{h1}}{1-\pi_{h3}} + \gamma_2 \frac{\pi_{h2}}{1-\pi_{h3}}. \tag{15}$$

It measures the shadow value of the Home-origin composition toward which Home region h substitutes when the tariff discourages purchases from ROW.

For ROW,

$$\mu_3 = \gamma_1 \frac{\pi_{31}}{1-\pi_{33}} + \gamma_2 \frac{\pi_{32}}{1-\pi_{33}}. \tag{16}$$

It measures the shadow value of ROW's sourcing composition across the same two Home origins, conditional on ROW importing from Home.

Regional weighting schemes. The tariff-substitution weights are

$$\boxed{\omega_h^S \equiv \frac{X_h \pi_{h3}(1-\pi_{h3})}{\sum_{\ell=1}^{2} X_\ell \pi_{\ell 3}(1-\pi_{\ell 3})}, \qquad h \in \{1, 2\}.} \tag{17}$$

For an interior equilibrium, these weights are nonnegative and satisfy

$$\omega_1^S + \omega_2^S = 1.$$

They measure each Home region's contribution to tariff-induced substitution away from ROW goods. The term

$$X_h \pi_{h3}(1-\pi_{h3})$$

combines the size of region h's expenditure with the sensitivity of its ROW import share to the common tariff.

In particular,

$$\frac{\partial \pi_{h3}}{\partial \log[1/(1+\tau)]} = \theta \pi_{h3}(1-\pi_{h3}). \tag{18}$$

The common factor θ cancels when constructing the normalized weights. Thus, ω_h^S identifies where the common tariff changes sourcing behavior most strongly.

The import-expenditure weights are

$$\boxed{\omega_h^M \equiv \frac{X_h \pi_{h3}}{\sum_{\ell=1}^{2} X_\ell \pi_{\ell 3}}, \qquad h \in \{1, 2\}.} \tag{19}$$

These weights are also nonnegative and satisfy

$$\omega_1^M + \omega_2^M = 1.$$

They measure each Home region's share of total tariff-inclusive expenditure on ROW goods. Because the common tariff rate is the same in both Home regions, the same weights also describe the regional distribution of tariff revenue.

The distinction between the two weights is therefore

$$\boxed{\omega_h^M \text{ identifies where Home imports and tariff revenue are located,}}$$

whereas

$$\boxed{\omega_h^S \text{ identifies where the tariff changes import behavior.}}$$

Writing all underlying objects explicitly, the regional-adjusted Home market-power index is

$$\boxed{\Psi_H^R = \underbrace{\frac{\sum_{h=1}^{2} \omega_h^S \left[\sum_{i=1}^{2} \gamma_i \frac{\pi_{hi}}{1-\pi_{h3}}\right]}{\sum_{i=1}^{2} \gamma_i \frac{\pi_{3i}}{1-\pi_{33}}}}_{\substack{\text{conditional-sourcing} \\ \text{component}}} \; \underbrace{\frac{\sum_{h=1}^{2} \omega_h^M \gamma_{xh}}{\sum_{h=1}^{2} \omega_h^S \gamma_{xh}}}_{\substack{\text{regional-incidence} \\ \text{component}}} .} \tag{20}$$

Comparison with the two-country benchmark. If the two Home regions can be aggregated exactly, the conditional sourcing composition generated by tariff-induced substitution coincides with ROW's conditional sourcing composition, and the two regional weighting schemes produce the same shadow valuation. In that case,

$$\mathcal{S}_H = 1, \qquad \mathcal{I}_H = 1, \qquad \Psi_H^R = 1. \tag{21}$$

The general formula then reduces to

$$\frac{\tau^*}{1+\tau^*} = \frac{1}{1+\theta\pi_{33}^*}, \tag{22}$$

which is equivalent to

$$\boxed{\tau^* = \frac{1}{\theta\pi_{33}^*}.} \tag{23}$$

This is the earlier two-country formula, with region 3 playing the role of the foreign country m.

In the general asymmetric three-region economy,

$$\Psi_H^R \neq 1.$$

The departure of Ψ_H^R from one is the exact Home-side correction generated by the inability to aggregate regions 1 and 2 into a single representative Home market.

Holding the ROW export response fixed,

$$\Psi_H^R > 1$$

raises the optimal tariff wedge relative to the aggregable-Home benchmark, whereas

$$\Psi_H^R < 1$$

lowers it.

Finally, because all starred trade shares, expenditures, wages, and multipliers are jointly determined with the optimal tariff, equation (2) is an exact equilibrium characterization of the optimal common tariff, rather than a reduced-form formula written only in primitive parameters.

1 Environment and assumptions

There is one tradable sector and labor is the only factor. Regions 1 and 2 form Home,

$$\mathcal{H} = \{1, 2\},$$

while region 3 is ROW. Region i has labor L_i, wage w_i, technology T_i, and all bilateral trade shares share the Eaton–Kortum elasticity $\theta > 0$.

The notation ni means that destination n purchases from origin i. The bilateral trade wedge is

$$\kappa_{ni} = d_{ni}(1 + \tau_{ni}). \tag{24}$$

The internal trade cost is symmetric,

$$d_{12} = d_{21} = d,$$

own trade costs satisfy $d_{11} = d_{22} = d_{33} = 1$, and the international iceberg costs

$$d_{13},\ d_{23},\ d_{31},\ d_{32} > 0$$

are exogenous and need not be symmetric.

Home imposes one common external tariff on imports from ROW,

$$\tau_{13} = \tau_{23} = \tau, \qquad \tau_{ni} = 0 \quad \text{otherwise}. \tag{25}$$

ROW does not retaliate. Tariff revenue is rebated locally to the Home region that collects it.

The bilateral Eaton–Kortum price component is

$$\boxed{\Phi_{ni} = T_i^{-1/\theta} \left[\gamma_{\mathrm{EK}}\, d_{ni}(1 + \tau_{ni}) w_i\right],} \tag{26}$$

where γ_{EK} is the standard Eaton–Kortum price-index constant. The destination price index and bilateral expenditure share are

$$P_n = \left[\sum_{i=1}^{3} \Phi_{ni}^{-\theta}\right]^{-1/\theta}, \qquad \boxed{\pi_{ni} = \frac{\Phi_{ni}^{-\theta}}{\sum_{k=1}^{3} \Phi_{nk}^{-\theta}}.} \tag{27}$$

Thus, π_{ni} is destination n's expenditure share on goods produced in region i.

Home-region tariff revenue is

$$R_h = \frac{\tau}{1+\tau} X_h \pi_{h3}, \qquad h \in \{1,2\}. \tag{28}$$

The factor $\tau/(1+\tau)$ is the share of tariff-inclusive import expenditure collected as revenue. It is the policy wedge that appears naturally in the government budget constraints.

The expenditure-income conditions are

$$X_h = w_h L_h + \frac{\tau}{1+\tau} X_h \pi_{h3}, \qquad h \in \{1,2\}, \tag{29}$$

$$X_3 = w_3 L_3. \tag{30}$$

Goods produced in each Home region clear according to

$$w_i L_i = \sum_{n=1}^{3} X_n \pi_{ni}, \qquad i \in \{1,2\}. \tag{31}$$

ROW goods-market clearing is

$$w_3 L_3 = \frac{1}{1+\tau} \sum_{h=1}^{2} X_h \pi_{h3} + X_3 \pi_{33}. \tag{32}$$

The Home government maximizes the sum of real absorption in its two regions,

$$\mathcal{W}_H = \frac{X_1}{P_1} + \frac{X_2}{P_2}. \tag{33}$$

A nominal normalization such as $P_1 = 1$ is imposed only after taking first-order conditions.

2 Constrained planner and useful derivatives

I retain all three expenditure-income conditions and the two Home goods-market-clearing conditions. The ROW goods-market-clearing equation is omitted from the Lagrangian by Walras' law, although it continues to hold in equilibrium.

Let γ_{xn} be the multiplier on region n's expenditure-income condition and let γ_i be the multiplier on the market-clearing condition for goods produced in Home region $i \in \{1,2\}$. The Lagrangian is

$$\begin{aligned} \mathcal{L} = \frac{X_1}{P_1} + \frac{X_2}{P_2} + \sum_{h=1}^{2} \gamma_{xh} \left[X_h - w_h L_h - \frac{\tau}{1+\tau} X_h \pi_{h3} \right] \\ + \gamma_{x3}(X_3 - w_3 L_3) + \sum_{i=1}^{2} \gamma_i \left[w_i L_i - \sum_{n=1}^{3} X_n \pi_{ni} \right]. \end{aligned} \tag{34}$$

For the policy derivation, define

$$q \equiv \frac{1}{1+\tau}, \qquad 1 - q = \frac{\tau}{1+\tau}. \tag{35}$$

For each Home destination $h \in \{1,2\}$,

$$\frac{\partial \log P_h}{\partial \log q} = -\pi_{h3}, \tag{36}$$

$$\frac{\partial \pi_{h3}}{\partial \log q} = \theta\pi_{h3}(1-\pi_{h3}), \tag{37}$$

$$\frac{\partial \pi_{hi}}{\partial \log q} = -\theta\pi_{hi}\pi_{h3}, \qquad i \in \{1,2\}. \tag{38}$$

For any origin k,

$$\frac{\partial \log P_n}{\partial \log w_k} = \pi_{nk}, \qquad \frac{\partial \pi_{ni}}{\partial \log w_k} = -\theta\pi_{ni}\left(\mathbf{1}_{\{i=k\}} - \pi_{nk}\right). \tag{39}$$

3 Step 1: the reduced common-tariff condition

The expenditure first-order condition for Home region h is

$$\frac{1}{P_h} + \gamma_{xh}\left[1-(1-q)\pi_{h3}\right] - \sum_{i=1}^{2}\gamma_i\pi_{hi} = 0, \qquad h \in \{1,2\}. \tag{40}$$

Using (36)–(38), the first-order condition with respect to $\log q$ is

$$\begin{aligned} 0 = &\sum_{h=1}^{2}\frac{X_h}{P_h}\pi_{h3} + \sum_{h=1}^{2}\gamma_{xh}X_h\pi_{h3}\left[q-(1-q)\theta(1-\pi_{h3})\right] \\ &+\theta\sum_{i=1}^{2}\gamma_i\sum_{h=1}^{2}X_h\pi_{hi}\pi_{h3}. \end{aligned} \tag{41}$$

Multiply (40) by $X_h\pi_{h3}$ and sum over the two Home regions:

$$\begin{aligned} \sum_{h=1}^{2}\frac{X_h}{P_h}\pi_{h3} = &-\sum_{h=1}^{2}\gamma_{xh}X_h\pi_{h3}\left[1-(1-q)\pi_{h3}\right] \\ &+\sum_{i=1}^{2}\gamma_i\sum_{h=1}^{2}X_h\pi_{hi}\pi_{h3}. \end{aligned} \tag{42}$$

Substituting (42) into (41) and collecting terms produces the reduced policy condition

$$\boxed{\frac{\tau}{1+\tau}\sum_{h=1}^{2}\gamma_{xh}X_h\pi_{h3}(1-\pi_{h3}) = \sum_{i=1}^{2}\gamma_i\sum_{h=1}^{2}X_h\pi_{hi}\pi_{h3}.} \tag{43}$$

This equation has a direct interpretation. The left-hand side is the tariff wedge multiplied by the shadow value of tariff-induced changes in the two regional expenditure-income constraints. The right-hand side is the shadow value of the tariff-induced reallocation of demand toward goods produced in Home regions 1 and 2.

Using (43) again in (42) gives a second useful relation:

$$\boxed{\sum_{h=1}^{2} \frac{X_h}{P_h}\pi_{h3} = -\frac{1}{1+\tau}\sum_{h=1}^{2}\gamma_{xh}X_h\pi_{h3}.} \tag{44}$$

The left-hand side is Home's direct real-welfare exposure to ROW goods. The right-hand side expresses the same exposure through the shadow values of the regional expenditure constraints.

4 Step 2: the ROW producer-price response

The X_3 first-order condition is

$$\gamma_{x3} = \sum_{i=1}^{2}\gamma_i\pi_{3i}. \tag{45}$$

The first-order condition with respect to $\log w_3$ is

$$\begin{aligned} 0 = & -\sum_{h=1}^{2}\frac{X_h}{P_h}\pi_{h3} + \theta\frac{\tau}{1+\tau}\sum_{h=1}^{2}\gamma_{xh}X_h\pi_{h3}(1-\pi_{h3}) - \gamma_{x3}w_3L_3 \\ & - \theta\sum_{i=1}^{2}\gamma_i\left[\sum_{h=1}^{2}X_h\pi_{hi}\pi_{h3} + X_3\pi_{3i}\pi_{33}\right]. \end{aligned} \tag{46}$$

The Home-demand terms in (46) cancel after using the reduced policy condition (43). Then, using (45) and $X_3 = w_3L_3$, the ROW producer-price condition becomes

$$\boxed{\sum_{h=1}^{2}\frac{X_h}{P_h}\pi_{h3} = -w_3L_3(1+\theta\pi_{33})\sum_{i=1}^{2}\gamma_i\pi_{3i}.} \tag{47}$$

The term $1+\theta\pi_{33}$ is the response of ROW producer-value exports to the ROW producer price. Define ROW producer-value exports to Home by

$$E_{H3} \equiv \frac{1}{1+\tau}\sum_{h=1}^{2}X_h\pi_{h3}. \tag{48}$$

Using ROW goods-market clearing (32) and $X_3 = w_3L_3$,

$$E_{H3} = w_3L_3(1-\pi_{33}). \tag{49}$$

Holding w_1, w_2, and q fixed,

$$\boxed{\varepsilon_3^V \equiv \frac{\partial \log E_{H3}}{\partial \log w_3} = 1+\theta\pi_{33}.} \tag{50}$$

The 1 is the mechanical effect of a higher producer price on export value, while $\theta\pi_{33}$ is the substitution effect through ROW's domestic expenditure share. This is an elasticity of producer-value exports, not a physical export quantity.

5 Step 3: isolate the regional-adjusted market-power force

For each destination $n \in \{1,2,3\}$, define the shadow value of its conditional sourcing composition across the two Home origins:

$$\mu_n \equiv \sum_{i=1}^{2} \gamma_i \frac{\pi_{ni}}{1-\pi_{n3}}. \tag{51}$$

Because $\pi_{n1} + \pi_{n2} = 1 - \pi_{n3}$, the ratio

$$\frac{\pi_{ni}}{1-\pi_{n3}}$$

is destination n's sourcing share from Home origin i, conditional on purchasing a Home good. Hence μ_n assigns market-clearing shadow values to destination n's composition of purchases across Home regions 1 and 2.

Define two sets of Home-region weights:

$$\omega_h^S \equiv \frac{X_h \pi_{h3}(1-\pi_{h3})}{\sum_{\ell=1}^{2} X_\ell \pi_{\ell 3}(1-\pi_{\ell 3})}, \tag{52}$$

$$\omega_h^M \equiv \frac{X_h \pi_{h3}}{\sum_{\ell=1}^{2} X_\ell \pi_{\ell 3}}, \qquad h \in \{1,2\}. \tag{53}$$

The weights ω_h^S measure where the common tariff changes import behavior; ω_h^M measure where Home's expenditure on ROW goods occurs.

Equation (43) becomes

$$\frac{\tau}{1+\tau} = \frac{\sum_{h=1}^{2} \omega_h^S \mu_h}{\sum_{h=1}^{2} \omega_h^S \gamma_{xh}}. \tag{54}$$

Next, combine the Home exposure equation (44), the ROW producer-price equation (47), and ROW goods-market clearing. This gives

$$\boxed{\sum_{h=1}^{2} \omega_h^M \gamma_{xh} = (1+\theta\pi_{33})\mu_3.} \tag{55}$$

Equation (55) is the bridge between the Home-side policy condition and the foreign producer-price response.

Define the regional-adjusted Home market-power index as

$$\boxed{\Psi_H^R \equiv \underbrace{\frac{\sum_{h=1}^{2} \omega_h^S \mu_h}{\mu_3}}_{\text{conditional-sourcing component}} \underbrace{\frac{\sum_{h=1}^{2} \omega_h^M \gamma_{xh}}{\sum_{h=1}^{2} \omega_h^S \gamma_{xh}}}_{\text{regional-incidence component}}.} \tag{56}$$

The first component compares the Home-production composition generated by tariff-induced substitution inside Home with ROW's composition of purchases from the two Home origins. The second compares where Home imports from ROW occur with where the tariff produces the strongest import-demand response, after weighting by the regional expenditure multipliers.

Combining (54), (55), and (56) gives the exact optimal-policy formula

$$\boxed{\frac{\tau^*}{1+\tau^*} = \frac{\Psi_H^{R,*}}{1+\theta\pi_{33}^*} = \frac{\Psi_H^{R,*}}{\varepsilon_3^{V,*}}.} \tag{57}$$

Thus,

$$\boxed{\text{optimal tariff wedge} = \text{regional-adjusted Home market power} \times \text{inverse ROW export response.}} \tag{58}$$

Equivalently, the tariff rate itself is

$$\boxed{\tau^* = \frac{\Psi_H^{R,*}}{1+\theta\pi_{33}^* - \Psi_H^{R,*}}.} \tag{59}$$

All starred objects are evaluated at the optimal equilibrium, so this is an exact fixed-point characterization rather than a reduced-form expression in primitive parameters alone.

6 Relation to the two-country benchmark

If the two Home regions can be aggregated exactly, the two regional corrections in (56) jointly satisfy

$$\Psi_H^R = 1.$$

The general three-region formula then collapses to

$$\frac{\tau^*}{1+\tau^*} = \frac{1}{1+\theta\pi_{33}^*}, \qquad \boxed{\tau^* = \frac{1}{\theta\pi_{33}^*}.} \tag{60}$$

This is exactly the earlier two-region formula (1), with region 3 playing the role of the foreign country m.

In the general asymmetric case, $\Psi_H^R \neq 1$. The internal trade cost affects the optimal external tariff through two channels:

$$d \longrightarrow \text{equilibrium relative wages} \longrightarrow \pi_{33} \longrightarrow 1+\theta\pi_{33},$$
$$d \longrightarrow \text{regional sourcing, import exposure, and tariff incidence} \longrightarrow \Psi_H^R.$$

The first is the conventional foreign export-response channel. The second is the new internal-regional channel.