Demographics, Trade, and Growth

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Introduction

Motivation

- In recent decades(70s-20), developing countries that experienced an increase in their working-age population also saw increases in trade and real GDP per capita ▸ Detail
  - Usually specialize in and export labor-intensive goods (Hanson, 2020)
  - Examples: China, Vietnam, Philippines and more
- However, in recent years, as the working-age pop. such as China and many other countries has declined, growth also slowed down
  - All else being equal, this encourages a shift in comparative advantage to capital-intensive goods

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Research Question: How much does demographic structure influence changes in trade patterns and economic growth?

Introduction

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Potential mechanisms

- Productivity (Age-varying ability in generating new ideas)
- Capital accumulation (Age-varying saving behavior)
- Trade determined by comparative advantage (CA) reallocates the production across countries and sectors
  - Ricardian CA: Difference in Productivity
  - Hecksher-Ohlin CA: Differences in K/L ratio

Job Market Research

- Empirical evidence ▸ Detail
  - Panel regression: higher working age share is related to higher
    - ★ Productivity growth
    - ★ Investment share of GDP, and growth rate of K/L ratio
  - Panel regression: lower trade cost is related to higher growth rate of K/L ratio
  - VARX model: The hump shape for IRF of $+1\%$ young people share shock on
    - ★ Productivity growth, and growth rate real capital stock per person
- Develop and calibrate a dynamic OLG trade model features
  - Demographic-induced productivity change
  - Demographic-induced capital accumulation
  - Trade based on Ricardian and Heckscher-Ohlin CA
- Implementation: China's development over 1981-2020, and do model-based projection for 2021-2060. ▸ Detail

Today's focus

Introduce the Model

- The demographic structure can influence
  - The knowledge stock (productivity)
  - The capital stock
- Trade based on Ricardian and Heckscher-Ohlin CA

Numerical experiments

- Close economy
  - Compares two steady states
    - Exogenous variable: age-varying survival rates and fertility rates
  - Examines the transition dynamics

Model

Overview

- Multi-country, multi-sector OLG model with Eaton-Kortum type trade
- Demographic structure influence
  - Productivity change
  - Capital accumulation
- Trade determined by comparative advantage regulates the allocation of production and changes trade patterns
  - Ricardian comparative advantage
  - Heckscher-Ohlin comparative advantage

Model

Production: production function

$$y_{n,t}^{j}(\omega) \equiv z_{n,t}^{j}(\omega)\left[N_{n,t}^{j}(\omega)^{\beta_n^j} K_{n,t}^{j}(\omega)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} m_{n,t}^{k,j}(\omega)^{\gamma_n^{k,j}} \tag{1}$$

- Intermediate good $\omega \in [0,1]$ from country n sector j: $y_{n,t}^{j}(\omega)$ are produced by labor, capital, and sectoral composite intermediate good
- Variety-specific productivity $z_{n,t}^{j}(\omega)$ drawn from Fréchet $F_{n,t}^{j}(z) = exp(-\lambda_{n,t}^{j} z^{-\theta})$
  - θ controls the variance of productivity distribution
  - $\lambda_{n,t}^{j}$ controls the mean of productivity for country n, sector j, at time t
    - ★ a.k.a knowledge stock

Model

Production: Knowledge stock dynamics (1/2)

(omit sector j and country n subscript for simplify)

- The mean of ideas arrived per period, α_t, determines technology stock (λ_t) dynamics (Oberfield and Buera, 2019):

$$\lambda_{t+1} - \lambda_t = \Gamma(1-\rho)\alpha_t(\lambda_t)^\rho \tag{2}$$

- New assumption: the age-dependent ability to generate new ideas

$$\lambda_{t+1} - \lambda_t = \Gamma\left(1-\rho\right)\left(\sum_g \eta_g \bar{N}_{g,t}\right)^\varphi N^\varphi \left(\lambda_t\right)^\rho ; \qquad \alpha_t \equiv \left(\sum_g \eta_g N_{g,t}\right)^\varphi \tag{3}$$

  - η_g: mean of ideas arrived per age g people per period
  - $N_{g,t}, \bar{N}_{g,t}$: number and share of age g people at time t
  - $\varphi < 1$: reflect some crowding effects, or duplication of idea
  - $\rho \in (0,1)$: capture the effects of existing knowledge stock
  - $\left\{\sum_g \eta_g N_{g,t}\right\}^\varphi$: total number of efficient idea per unit of time

Model

Production: Knowledge stock dynamics (2/2)

- Technology stock (λ_t) dynamics:

$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = \Gamma\,(1-\rho)\,(\sum_g \eta_g \bar{N}_{g,t})^{\varphi} N^{\varphi}\,(\lambda_t)^{\rho-1} \tag{4}$$

- At steady state, given population growth at rate $1+g_n$, knowledge stock growth at $(1+g_n)^{\frac{\varphi}{1-\rho}}$

- W/o demographic effects, isomorphic to Semi-endogenous growth framework (Chad Jones, 2022)

$$\frac{A_{t+1} - A_t}{A_t} = cN_t^{\varphi} A_t^{\rho-1}; \quad \rho < 1 \tag{5}$$

  - A_t: level of productivity; counterpart in my model is ${\lambda_t}^{1/\theta}$: mean parameter of productivity for varieties $\omega \in [0,1]$
    - ★ θ controls the variance of the productivity distribution
  - $A_t \sim \lambda_t^{1/\theta}$ implies $\Delta \log A_t \sim 1/\theta \Delta \log \lambda_t$

Model

Households (1/2)

(Omit country subscripts for simplicity)

- Households work at age 16, retired at age 65 and die at age $G = 100$
- During the working age, households own 1 unit of labor endowment
- There exogenous variable governing the demographic process
  - The initial number of population across ages: N_{g,t_0}
  - The fertility rate of age g households at time t: $f_{g,t}$
  - The probability of surviving to age g at time t: $S_{g,t}$
- The age g households that was born in period t choose lifetime consumption $\{c_{g,t+g-1}\}_{g=1}^{G}$ and savings $\{a_{g+1,t+g}\}_{g=1}^{G-1}$ to maximize expected lifetime utility

$$\sum_{g=1}^{G} \beta^{g-1} \psi_{n,t+g-1} S_{g,t+g-1} u\left(c_{g,t+g-1}\right)$$

  - $u(c) = (c^{1-1/\sigma})/(1-1/\sigma)$
  - ψ_t saving frictions, capture other forces impact on saving behavior

Model

Households (2/2)

The budget constraint for households at age $g \in [1, G]$, time t is

$$P_{C,t} c_{g,t} + P_{I,t} a_{g+1,t+1} = P_{I,t} (1 + r_t) a_{g,t} + W_t (1 - \tau_t^L) E_t l_g + ts_t^D + ts_t^T$$
$$\forall\, t:\ a_{1,t} = a_{G+1,t} = 0$$

- $P_{C,t}$ and $P_{I,t}$: price level for consumption or investment
- W_t and R_t: wage and rental rate
- Households save or borrow in the quantity of $a_{n,g+1,t+1}$ under interest rate ▸ Detail

$$r_{t+1} = \frac{R_{t+1}}{P_{I,t+1}} - \delta$$

- Household at age g own labor endowment $l_g = 1$, $\forall g \in [16, 65]$
- labor supply is adjusted for labor supply frictions τ_t^L and human capital index $E_{n,t}$
- Transfers are equally distributed across the households
  - ts_t^D is the trade deficit induced transfer (Caliendo et.al, 2018) ▸ Detail
  - ts_t^T accidental death induced transfer: saving left by households who die before age G ▸ Detail

Model

Trade

(I omit time t subscript to simplify notation)

- "Iceberg" trade costs: $\kappa_{ni}^{j} \geq 1$ for country n by sector j goods from country i
- Following Eaton and Kortum (2002), the fraction of country n's expenditures in sector j goods source from country i is:

$$\pi_{ni}^{j} = \frac{\lambda_i^j \left(c_i^j \kappa_{ni}^j\right)^{-\theta}}{\sum_{i=1}^{N} \lambda_i^j \left(c_i^j \kappa_{ni}^j\right)^{-\theta}} \tag{6}$$

  - c_n^j is the unit price of an input bundle in country n sector j

$$c_n^j \equiv \Upsilon_n^j \left[(W_n)^{\beta_n^j} (R_n)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} P_n^{k\,\gamma_n^{k,j}} \tag{7}$$

    - ★ P_n^j is the price of sectoral composite goods from country n sector j

▸ Detail

Numerical experiments

- Close economy
  - Exogenous variable: age-varying survival rates and fertility rates
  - Compares several steady-state pairs
    - ★ High fertility rate v.s. low
    - ★ Low survival rate v.s. high
  - Examines the transition dynamics
    - ★ Population growth slows down: from high fertility rate to low
    - ★ People live longer: from low survival rate to high

Steady State

Exogenous variables

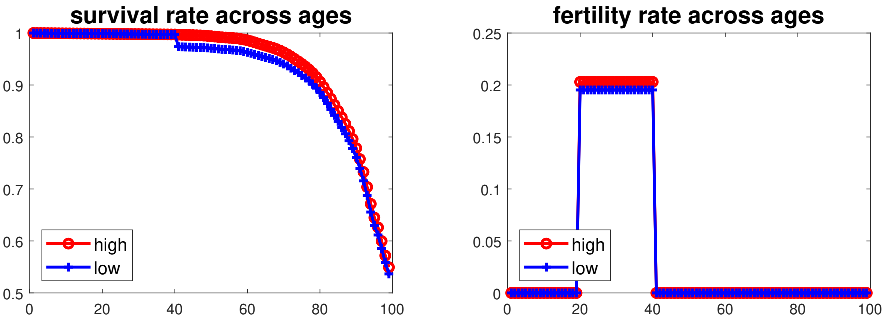


Figure: Exogenous variables

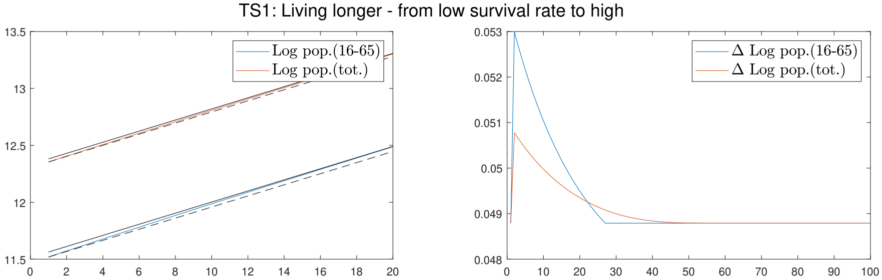
Compare Steady State

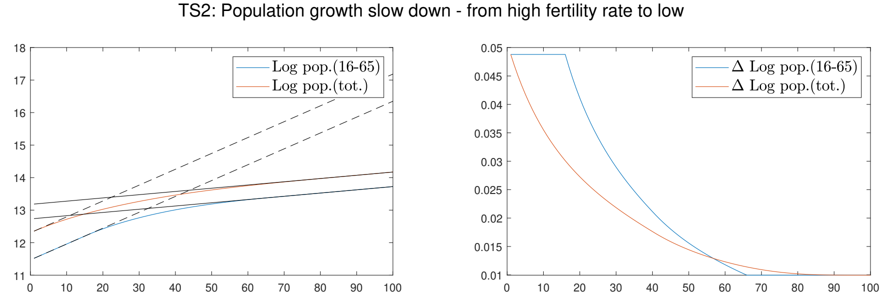
	Case control	Case 1	Case 2
Survival rate	low	**high**	low
Fertility rate	high	high	**low**
Average lifespan	60	**70.79**	60
Population growth	1.05	1.05	**1.01**
Implied TFP growth	1.0165	1.0165	**1.0033**
labor share	0.4338	0.4412	0.6394
Per efficient person			
Capital stock	1.1672	1.3516	2.5128
Output	0.6034	0.6408	1.009
Consumption	0.4446	0.4569	0.8204
Investment	0.1588	0.1839	0.1886
capital - efficient labor ratio	2.6906	3.0635	3.93
Price ratio			
Real wage rate	0.9272	0.9682	1.0521
Real rental rate	0.1723	0.158	0.1339

- Case 1 v.s. Control: A higher average lifespan increases savings, which, acting as a supply of capital, leads to higher capital per efficient person
- Case 2 v.s. Control: With slower population and TFP growth, the number of effective persons grows more slowly. Less capital used to be spread across individuals, leads to higher capital per efficient person

Transitional dynamics: Population changes over time ▸ detail

Shock at Time 1: Survival Rate or Fertility Rate shocks lead economy moving from S.S.0 to S.S.1





Transitional dynamics: knowledge stock, λ_t

$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = (\lambda_t)^{\rho-1} \left\{ \sum_g \eta_g N_{g,t} \right\}^{\varphi} \Gamma(1-\rho)$$

Simple application

- $\eta_g = c > 0$ if $g \in (16, 65)$ and $\eta_g = 0$ if $g \notin (16, 65)$
- Given $\lambda_{t_0} = 1$, if the technology process is on the balance growth path at $t = t_0$, one can calculate c from

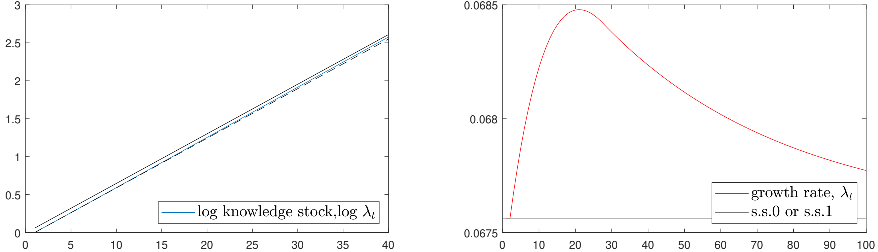
$$1 + g_{\lambda,t+1} = 1 + g_\lambda = N_t^{\varphi} (\lambda_t)^{\rho-1} \left\{ \sum_g \eta_g \bar{N}_{g,t} \right\}^{\varphi} \Gamma(1-\rho), \qquad t = t_0$$

Implication

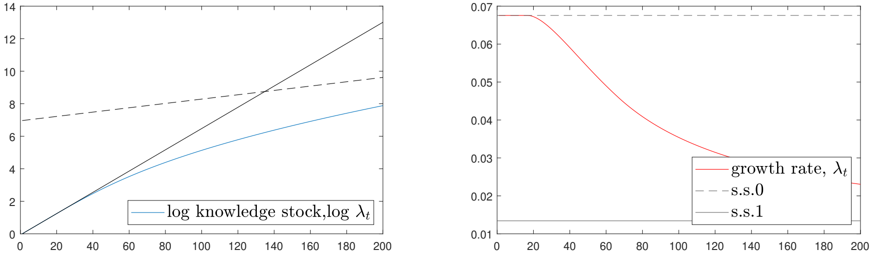
- In the short run, an increase in the level of working-age population leads to higher TFP growth.
- At steady state, given population growth at rate $1 + g_n$, knowledge stock growth at $(1+g_n)^{\frac{\varphi}{1-\rho}}$

Transitional dynamics: Knowledge stock changes over time

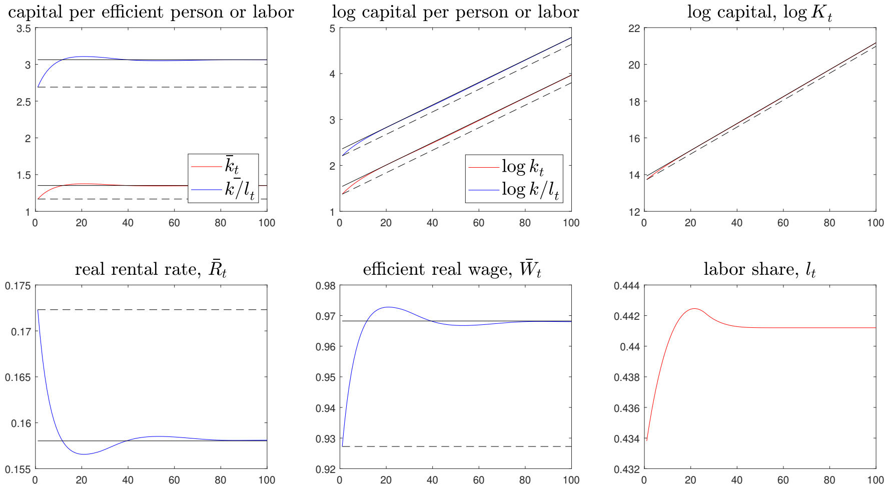
TS1: Living longer - from low survival rate to high



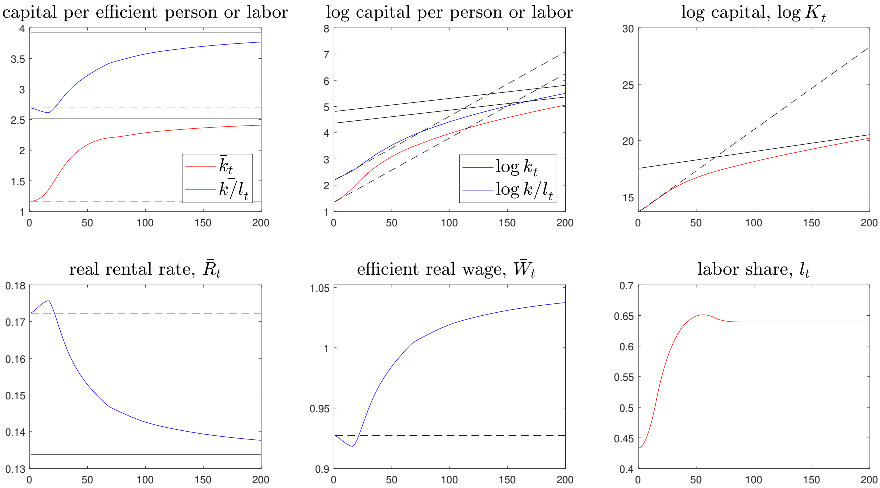
TS2: Population growth slow down - from high fertility rate to low



Transitional dynamics: TS1 - living longer



Transitional dynamics: TS2 - population growth slow down



Thank You

Transitional dynamics ▸ Back

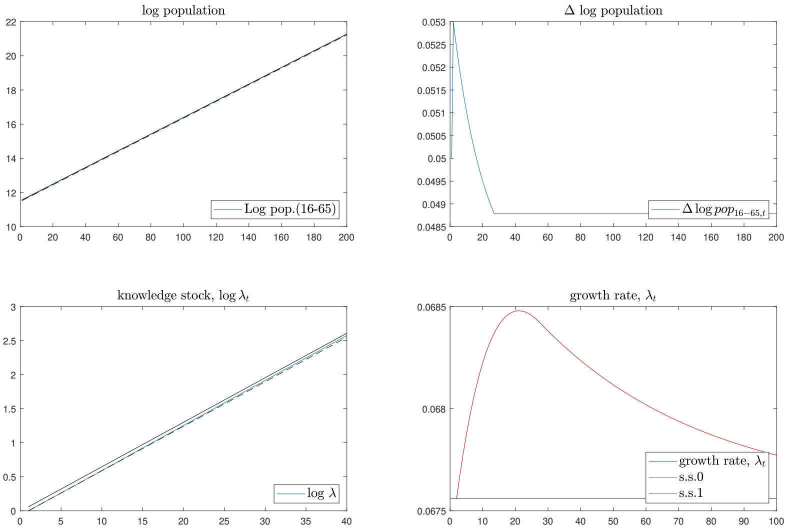


Figure: Live longer

Transitional dynamics ▸ Back

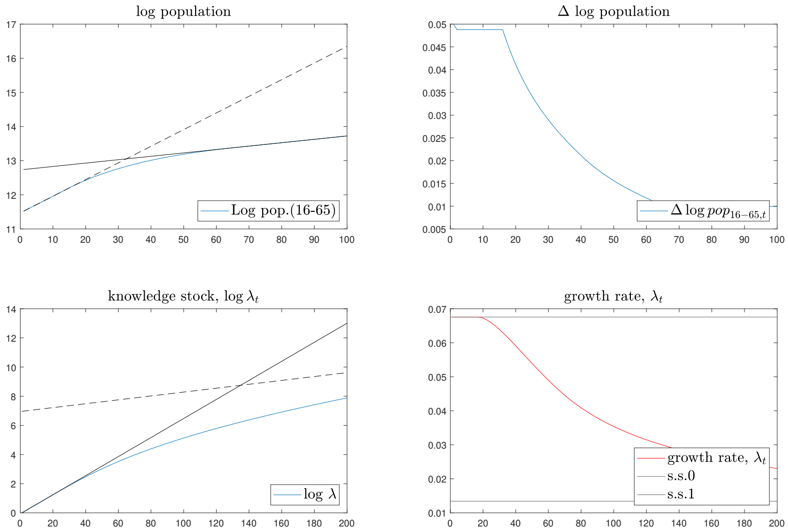


Figure: Growth slow down

Model

Aggregation ▸ Back

Capital

$$\sum_{j=1}^{J}\int_0^1 k_{n,t}^j(\omega)d\omega = K_{n,t} = \sum_{g=E+1}^{E+G}\eta_{n,g-1,t-1}a_{n,g,t} \tag{8}$$

Labor

$$\sum_{j=1}^{J}\int_0^1 l_{n,t}^j(\omega)d\omega = N_{n,t} = \sum_{g=E+1}^{E+G}\eta_{n,g,t}l_g \tag{9}$$

Consumption

$$C_{n,t} = \sum_{g=E+1}^{E+G}\eta_{n,g,t}c_{n,g,t} \tag{10}$$

Investment

$$I_{n,t} \equiv K_{n,t+1} - (1-\delta)K_{n,t} \tag{11}$$

▸ Steady State ▸ Transitional Dynamics

Model

Equilibrium ▸ Back

The model economy is summarized by time invariant parameters $(\beta_n^j, \gamma_n^{j,k}, \gamma_n^j, \alpha_{C,n}^j, \alpha_{I,n}^j, \delta,)$, time varing process of demographics process $\eta_{n,g,t}$, sectoral productivities λ_n^j, sectoral trade costs κ_{ni}^j, the initial capital and capital distribution: K_{n,t_0} and a_{n,g,t_0}, trade imbalances $\phi_{n,t}$ and discount factors $\phi_{n,t}$.

Definition

A competitive equilibrium of this model consists sequences of allocations $\forall g \in [E+1, E+G]$: $(a_{n,g,t}, c_{n,g,t}, C_{n,t}, K_{n,t}, I_{n,t}, \pi_{ni,t}^j)$ and prices $(P_{n,t}^j, R_{n,t}, W_{n,t})$ that satisfy the following conditions:

- The households taking prices transfer and deficit as given, optimize lifetime utility.
- Firms taking prices as given, minimize production cost.
- Each country purchases each variety from the least costly supplier/country.
- All markets are clear

Numerical experiments: Compare Steady State

Effect of Age structure and trade

Table: Compare steady state

Autarky	Baseline Case		Case 1	
	cty 1	**cty 2**	**cty 1**	**cty 2**
Aggregate Capital	14.20	14.20	**15.94**	14.20
Real income per person	0.39	0.39	**0.40**	0.39
Free Trade	**Baseline Case**		**Case 1**	
	cty 1	**cty 2**	**cty 1**	**cty 2**
Aggregate Capital	34.08	34.08	**43.91**	40.93
Real income per person	0.66	0.66	**0.73**	0.71
Balassa (1965) CA index Definition				
Sector 1 (capital-intensive)	1.0	1.0	**1.012**	0.988
Sector 2 (labor-intensive)	1.0	1.0	0.988	**1.012**

Under Autarky, Case 1 v.s. Baseline:

- Country 1 has more population ages 40-65, thus it has **more** aggregate capital supply in steady state

Under Free Trade, Case 1 v.s. Baseline:

- Country 1, sector 1 gain CA in the capital-intensive sector
- Trade amplifies welfare gains

Balassa (1965) Revealed comparative advantage (RCA) index

$$RCA_{nj} = \frac{\frac{Export_{n,j}}{\sum_n Export_{n,j}}}{\frac{\sum_j Export_{n,j}}{\sum_{j,n} Export_{n,j}}} \tag{12}$$

where n means country, j means sector, $Export_{n,j}$ means the value of country n's sector j exports.

- The higher RCA_{nj}, the higher degree of specialization for country n in sector j products.

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Empirical

Data source

The United Nations Statistics Division (UNSD)

- Age cohorts share for every 5 years, Dependence ratio, Old dependence ratio, Young dependence ratio, Total population

Penn World Table (PWT 10.01)

- Average annual hours worked by persons engaged, Number of persons engaged, Mean years of schooling, Capital stock, Real GDP, Average depreciation rate of the capital stock
- TFP calculated by PWT based on above variables

CEPII

- Imports and Exports between two countries

World Development Indicators (WDI)

- Share of household consumption, capital formation, government consumption (% share of GDP), residents new patents application, residents new industrial design application

Panel Regression

Effect of Demographic structure on TFP growth

Back

$$GRTFP_{it,t+4} = Constant + \alpha_1 Demographic_{it} + \alpha_2 Controls_{it} + f_i + f_t + \varepsilon_{it} \quad (13)$$

- 74 countries. I divide the entire period of 1970–2019 into 10 non-overlapping 5 year periods: period 1 (1970–1974), period 2 (1975–1979), period 3 (1980–1984), period 4 (1985–1989), period 5 (1990–1994),... and period 10 (2015–2019)
- i means country; t means year
- Dependent variables, $GRTFP_{it,t+4}$: Investment K/L Ratio
  - Average TFP growth rate during the period t to $t+4$
  - Average number of new patents applications during the period t to $t+4$
  - Average number of new industrial design applications during the period t to $t+4$
- $Demographic_t$: Working age share [15-64/total] (%) ; Share of people at different age intervals (%)
- $Controls$: Initial log real GDP per capita; f_i and f_t: fixed effects

Panel VARX model

Capital accumulation, TFP, and economic growth

VARX model:

$$Y_{n,t} = C + AY_{n,t-\mathbf{1}} + BX_{n,t} + \varepsilon_{n,t}$$

Endogenous variables:

$$Y_{nt} = \begin{bmatrix} the\ 5\ year\ growth\ rate\ of\ capital\ per\ person\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ TFP\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ the\ real\ GDP\ per\ capita\ (\%) \end{bmatrix}_{Country\ n, time\ t}$$

Exogenous variables: Demographic Structure (age shares):

$$X_{nt} = \begin{bmatrix} young\ people\ share\ (\%),\ (0-14) \\ old\ people\ share\ (\%),\ (65+) \\ trade\ cost\ change\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ population(\%) \end{bmatrix}_{Country\ n, time\ t}$$

Time interval: 1 unit of time = 5 years. e.g. t = 1 means first 5 years Back

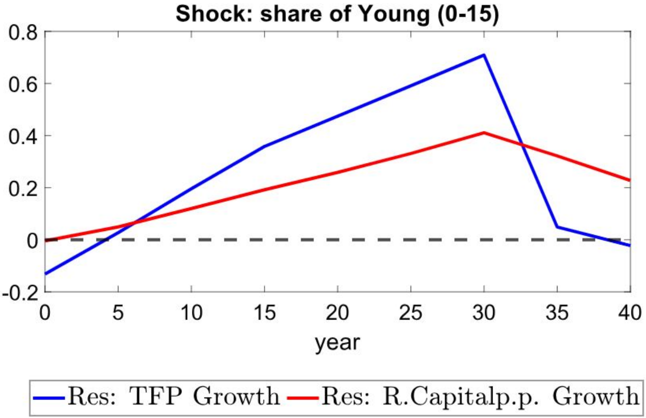
Panel Regression Results

	Average value in the future 4 years		
VARIABLES	TFP growth rate	Patent.Applications (per 1000 people)	Industrial.Design.Applications (per 1000 people)
(0-24)/ToT.	26.22***	-1.56***	-0.55***
	(4.24)	(-7.06)	(-3.87)
(25-49)/ToT.	34.48***	0.18	0.71***
	(4.28)	(0.46)	(2.87)
(50-74)/ToT.	43.60***	4.90***	1.08***
	(4.41)	(7.40)	(2.93)
(75+)/ToT.	13.47	-2.59	-1.85**
	(0.90)	(-1.59)	(-1.99)
Initial.Log.Dependent	-3.51***		
	(-4.49)		
Observations	732	395	215
R-squared	0.263	0.880	0.939
Time FE	YES	YES	YES
Country FE	YES	YES	YES

◂ Hump shape for the coefficients ▸ Back

Panel VARX model

IRF of exogenous shock on
I.TFP growth; **II**. Growth rate of real capital stock per person



The IRF of +1% young people share shock is hump shape

▸ Other IRFs ▸ Back

Panel regression

- TFP growth
  - Hump shape for the relation of age and TFP growth
    - ★ same for new patents application or new industry design application

VARX

- The IRF of $+1\%$ young people share shock is hump shape
  - Shock will pass down as people grow up

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Panel Regression

Effect of Demographic structure on investment and consumption

$$Ave.Y_{it,t+4} = Constant + \beta_1 Demographic_{it} + f_i + f_t + \varepsilon_{it} \tag{14}$$

- Y: Investment, or consumption share of GDP
- $Ave.Y_{it,t+4}$: Average investment, or consumption share of GDP during the period t to $t+4$:

$$Ave.Y_{it,t+4} = \sum_{s=t+0}^{t+4} \frac{Y_{i,s}}{5}$$

◂ Back

Panel Regression

Effects of demographic structure and trade cost change on capital/labor ratio

$$GR.K/L_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 TradeCost_{it} + \beta_3 Control_{it} + f_i + f_t + \varepsilon_{it} \quad (15)$$

- $GR.K/L_{it,t+4}$: Average capital per person (k) growth rate (%) for country i during the period t to $t+4$:

$$GR.K/L_{it,t+4} = [\frac{k_{i,s+4}}{k_{i,s}}]^{\frac{1}{4}} - 1$$

- $TradeCost_{it}$: The trade cost for country i at time t, which is constructed as the Head-Ries (HR) index (Head and Mayer, 2004):

$$TradeCost_{it} = (\frac{\pi_{i,row}}{\pi_{row,row}} \frac{\pi_{row,i}}{\pi_{ii}})^{-\frac{1}{2\theta}}$$

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Panel Regression Results

Robust: every non-overlapping 8 years

	Average value in the future 4 years		
VARIABLES	Cap.Formation(% GDP)	Gross.Consumption(% GDP)	K/L growth rate
(0-24)/ToT.	16.69***	92.44***	24.08***
	(2.64)	(14.84)	(4.50)
(25-49)/ToT.	29.11***	60.55***	39.21***
	(4.14)	(5.58)	(5.36)
(50-74)/ToT.	37.83**	59.95***	19.18
	(2.05)	(3.23)	(1.66)
(75+)/ToT.	-124.60***	150.74***	4.22
	(-2.77)	(3.21)	(0.24)
Trade Cost			-0.83**
			(-2.11)
Initial.Log.Dependent			-1.98***
			(-3.21)
PoP.Growth			-35.31**
			(-2.08)
Observations	724	725	758
R-squared	0.972	0.996	0.787
Time FE	YES	YES	YES
Country FE	YES	YES	YES

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Motivation

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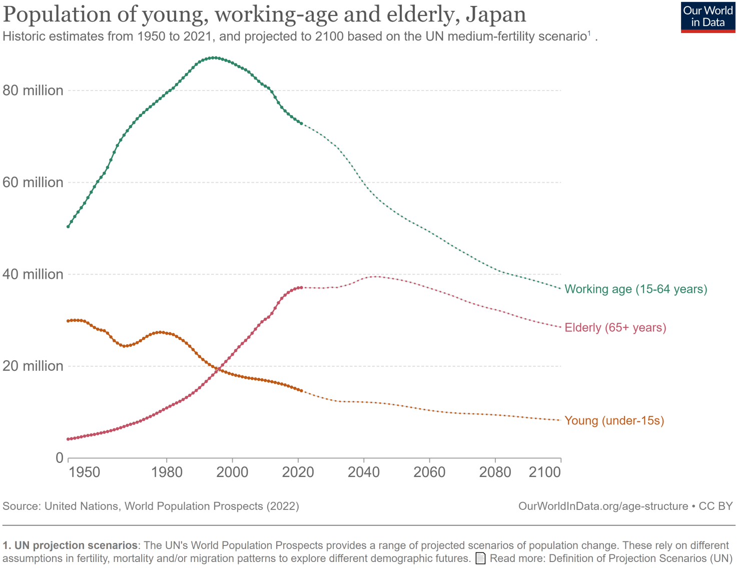
Motivation



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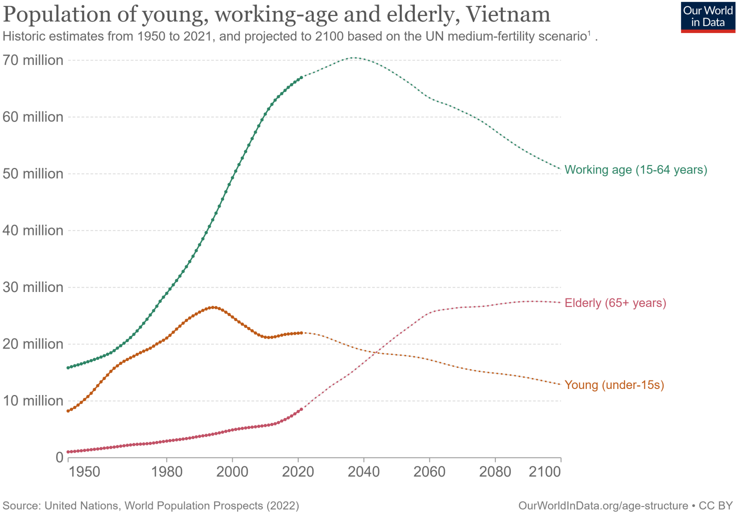
Motivation



1. UN projection scenarios: The UN's World Population Prospects provides a range of projected scenarios of population change. These rely on different assumptions in fertility, mortality and/or migration patterns to explore different demographic futures. Read more: Definition of Projection Scenarios (UN)

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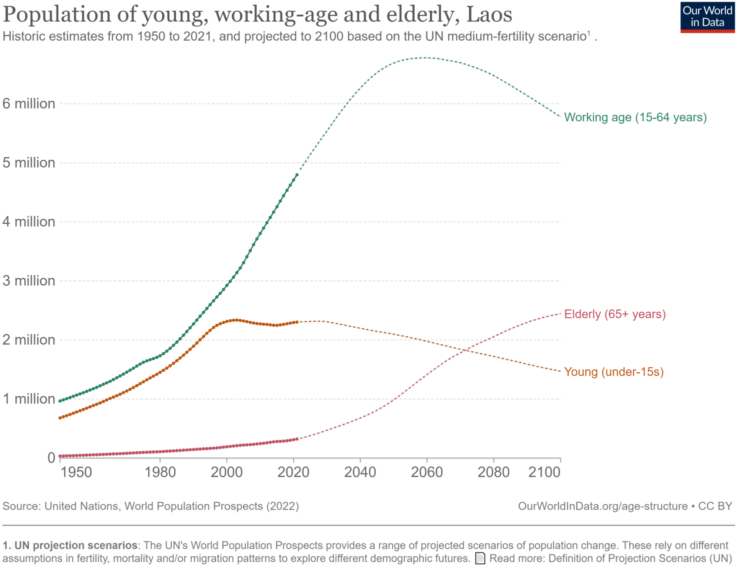
Motivation



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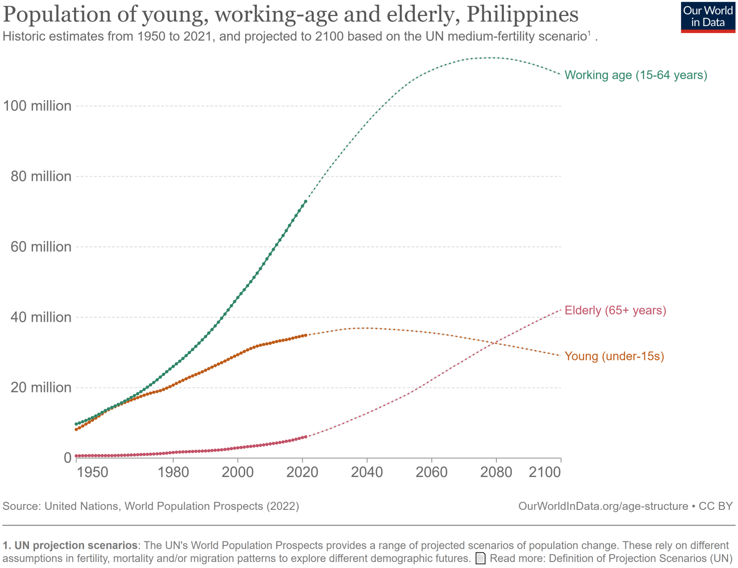
Motivation



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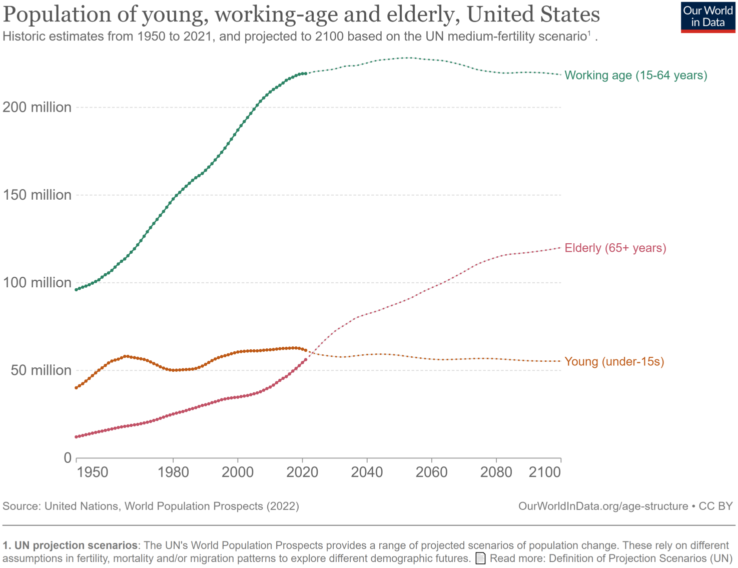
Motivation



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Motivation

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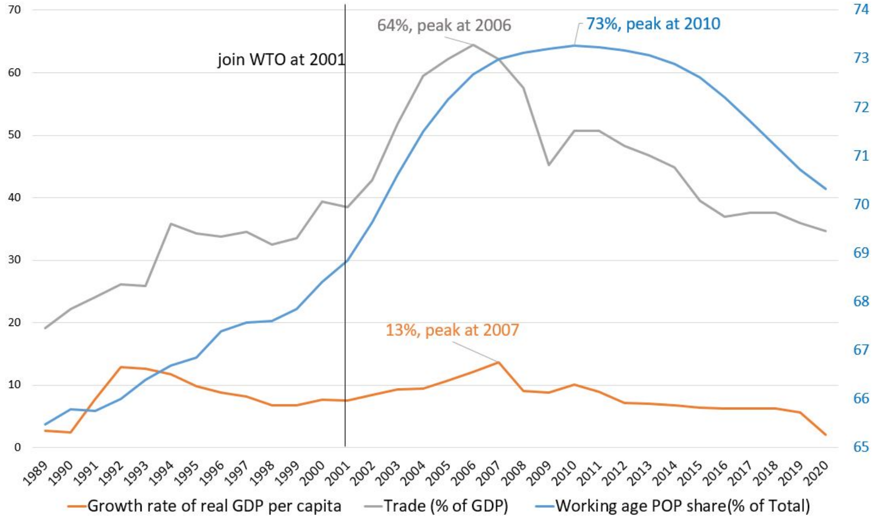
Motivation



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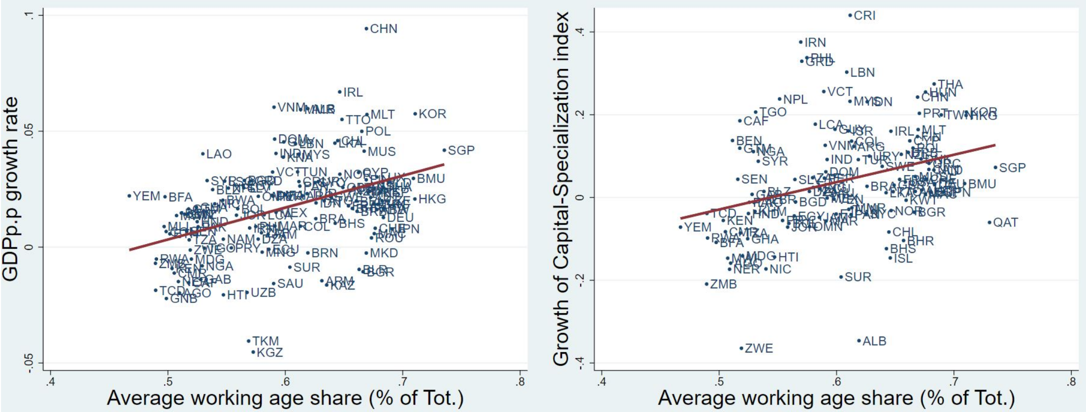
Motivation



▸ Detail ◂ Back

Motivation

Capital-Specialization Index

$$Capital - Specialization\ Index_{n,t} = \sum_{j=1}^{J} \frac{Export_{n,t}^{j}}{\sum_{j=1}^{J} Export_{n,t}^{j}} \cdot CI^{j}$$

- CI^j: capital intensive index of sector j
  - $CI^j = 1$ means sector j is capital intensive sector
  - $CI^j = 0$ means sector j is not capital intensive sector

- A sector is capital intensive sector if
  - Capital valued added share > mean (across all sector) of capital valued added shares

- $Export_n^j$: Total exports of country n for sector j goods

Back

Facts of China

Growth slow down, and Old before rich :

- Real GDP per capita growth trended down since 2008
- Working-age share trended down since 2010
- At 2021
  - Median age CHN v.s. USA: 37.9 v.s 37.7
  - Real GDP per capita: CHN is 28% of USA

Past: Large and growing working-age share & openness to trade

- Low level of wages, specialize in labor-intensive goods
- Demographic-induced TFP growth
- Growing capital accumulation from working age people

Question: How do demographics affect trade and growth of China in the past and future?

Back

Demographics and TFP

Table 2: The effect of demographic structure on technology change

VARIABLES	Average TFP growth rate in the future 7 years					
Initial.ln.RGDP.p.c	-2.78***	-0.17**	-1.96***	-2.93***	-0.18***	-2.19***
	(-4.32)	(-2.53)	(-4.66)	(-4.55)	(-2.64)	(-4.92)
Dep.Ratio [0-14, 65+]/[15-64]	-2.11*	-2.58***	-5.32***			
	(-1.88)	(-3.90)	(-4.89)			
Work.Share [15-64]/ToT				8.31***	7.61***	17.12***
				(2.76)	(4.20)	(5.50)
Constant	25.75***	3.48***	20.85***	20.69***	-2.80***	9.08***
	(4.44)	(3.37)	(4.83)	(3.53)	(-3.91)	(2.88)
Observations	439	439	439	439	439	439
R-squared	0.361	0.090	0.271	0.367	0.091	0.280
Time FE	YES	YES	NO	YES	YES	NO
Country FE	YES	NO	YES	YES	NO	YES

Robust t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1

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Demographics and TFP

Table 4: The effect of demographic structure on technology change

VARIABLES	Average TFP growth rate in the future 7 years					
Initial.ln.RGDP.p.c	-2.86***	-2.63***	-2.92***	-3.11***	-2.66***	-3.10***
	(-4.37)	(-3.80)	(-4.17)	(-4.49)	(-3.71)	(-4.28)
Child.Dep.R [0-14]/[15-64]	-2.58**		-2.70**			
	(-2.05)		(-2.08)			
Old.Dep.R [65+]/[15-64]		0.93	2.45			
		(0.22)	(0.55)			
Child.Share[0-14]/ToT				-9.31***		-9.41***
				(-2.72)		(-2.80)
Old.Share [65+]/ToT					3.06	-1.02
					(0.41)	(-0.14)
Constant	26.51***	22.79***	26.77***	30.40***	22.96***	30.42***
	(4.42)	(3.80)	(4.30)	(4.53)	(3.73)	(4.56)
Observations	439	439	439	439	439	439
R-squared	0.363	0.355	0.364	0.370	0.355	0.370
Time FE	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES

Robust t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1

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Demographics and TFP

Table 7: The effect of demographic structure on Investment, Saving and Consumption

	Average value (% GDP) in the future 7 years			
VARIABLES	Dom.Saving	Cap.Formation	Fix.Cap.Formation	Consumption
Dep.Ratio [0-14, 65+]/[15-64]	-7.63	-9.79*	-9.80*	7.63
	(-1.26)	(-1.91)	(-1.94)	(1.26)
Constant	26.65***	28.48***	27.36***	73.35***
	(6.18)	(7.44)	(7.33)	(17.00)
Observations	432	431	427	432
R-squared	0.792	0.627	0.587	0.792
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Robust t-statistics in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

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Model

Financial Market

The financial market works with zero frictions

- Receive deposits of $\sum a_{g,t} N_{g,t}$ from individuals
  - Repay those individuals an amount $(1 + r_t) \sum a_{g,t} N_{g,t}$
- Loaned an amount $K_t = \sum a_{g,t} N_{g,t}$ to firms to use in production
  - Receives an amount $(1 + R_t - \delta) K_t$ from firms
- Market clear implies

$$r_t = R_t - \delta \tag{16}$$

Back

Model

Trade deficit-induced transfers

- A pre-determined share of GDP, $\phi_{n,t}$ is sent to a global portfolio, which in turn disperses a per-capita lump-sum transfer, T_t^P,to every country

- The net transfer, also recognized as trade deficit, are calculated as:

$$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) + \bar{L}_{n,t} T_t^P \tag{17}$$

- Dividing by the total economically relevant population $\bar{L}_{n,t}$ implies that total bequests are equally distributed across the population

$$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) + \frac{\bar{L}_{n,t}}{\sum_{n=1}^{N} \bar{L}_{n,t}} \sum_{n=1}^{N} \phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) \tag{18}$$

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Model

Demographics-induced transfers

- $TRSV_{n,t}$ is defined as demographic structure change-induced transfer which is due to the number of population changes between cohort $(s-1, t-1)$ and (s, t)

$$TRSV_{n,t} = P_{n,I,t}\,(1 + r_{n,t}) \sum_{g=E+2}^{E+S} (\eta_{n,g-1,t-1} - \eta_{n,g,t})\, a_{n,g,t} \tag{19}$$

  - The number of population change can either counted as net death $(\eta_{n,g-1,t-1} - \eta_{n,g,t} > 0)$ or net immigrant $(\eta_{n,g-1,t-1} - \eta_{n,g,t} < 0)$
  - The asset change due to net death is treated as positive bequests
  - The net immigrant (g, t) enter country n with zero assets, and is treated as negative bequests

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Model

Steady State

Definition 1 (Steady-state equilibrium): A steady-state equilibrium in the perfect foresight overlapping generations trade model is defined as constant allocations of stationary consumption, saving and prices: $\left\{\{\bar{c}_{n,g}\}_{g=E+1,\ n=1}^{E+G,\ N},\ \left\{\bar{b}_{n,g+1}\right\}_{g=E+1,\ n=1}^{E+G-1,N},\ \left\{\bar{W}_n,\ \bar{R}_n\right\}_{n=1}^{N}\right\}$, such that:

- The households taking prices, transfer as given, optimize lifetime utility
- Firms taking prices as given, minimize production cost
- Each country purchases each variety from the least costly supplier/country
- The population distribution is not changed with time and defined as stationary steady-state distribution $\left\{\eta\star_{n,g}\right\}_{g=1}^{E+G}$
- The growth rate of sectoral TFP in the steady state is assumed as zero, which implies that the demographic induced growth $g\left(\vec{\eta}_{n,t};\vec{\alpha}_{n,t}^{j}\right)$ will be balanced out by orthogonal shocks $\vartheta_{n,t}^{j}$
- All markets are clear

▸ Equations ▸ Back

Definition 2 (The transitional dynamics equilibrium): The transitional dynamics equilibrium in the perfect foresight overlapping generations trade model with exogenous population dynamics is defined as sequences of allocations $\{c_{n,g,t}\}_{g=E+1,\ n=1}^{E+G,\ N}$, $\{b_{n,g+1,t+1}\}_{g=E+1,\ n=1}^{E+G-1,N}$ and prices $\{W_{n,t},\ R_{n,t}\}_{n=1}^{N}$ such that:

- The households taking prices, and transfer as given, optimize lifetime utility
- Firms taking prices as given, minimize production cost
- Each country purchases each variety from the least costly supplier/country
- All markets are clear

Equations Back

Model

Steady State (1/2)

Table: Steady-state conditions (1/2)

H1	$L_n^\star \equiv \sum_{g=1}^{E+G} \eta_{n,g}^\star$; $\bar{L}_n^\star \equiv \sum_{g=E+1}^{E+G} \eta_{n,g}^\star$; $N_n^\star = \left(1-\tau_n^{\star L}\right) \sum_{g=E+1}^{E+G_0} \eta_{n,g}^\star l_g = \left(1-\tau_n^{\star L}\right) \sum_{g=E+1}^{E+G} \eta_{n,g}^\star l_g$	$\forall(n)$
H2	$P_{n,C}^\star c_{n,g}^\star + P_{n,I}^\star b_{n,g+1}^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) P_{n,I}^\star b_{n,g}^\star + W_n^\star \left(1-\tau_n^{\star L}\right) E_n^\star l_g + \frac{TRSV_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$ for $\forall g \geq E+1$	$\forall(n)$
H3	$b_{n,E+1}^\star = b_{n,E+G+1}^\star = 0$, $c_{n,E+g}^\star > 0$, $\left\{c_{n,g}^\star\right\}_{g=E+1}^{E+G}$, $\left\{b_{n,g+1}^\star\right\}_{g=E+1}^{E+G-1}$	$\forall(n)$
H4	$TRS^V{}_n^\star = \left(\frac{R_n^\star}{P_{n,I}^\star}\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$; $TRS^D{}_n^\star = (1-\delta) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$	$\forall(n)$
H5	$TRSV_n^\star = P_{n,I}^\star \left(TRS^V{}_n^\star + TRS^D{}_n^\star\right) = P_{n,I}^\star \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$	$\forall(n)$
H6	$\left(\frac{c_{n,g+1}^\star}{c_{n,g}^\star}\right)^{1/\sigma} = \beta\left(\frac{\psi_{n,t+1}}{\psi_{n,t}}\right) \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right)$ for $\forall\ g \in [E+1, E+G-1]$	$\forall(n)$
H7	$C_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star c_{n,g}^\star$; $I_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star i_{n,g}^\star$; $K_n^\star = \sum_{g=E+1}^{E+G} \eta_{n-1,g-1}^\star b_{n,g}^\star$	$\forall(n)$
F1	$W_n^\star E_n^\star N_n^\star = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$
F2	$R_n^\star K_n^\star = \sum_{j=1}^{J} \left(1-\beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$

Model

Steady State (2/2)

Table: Steady-state conditions (2/2)

F3	$X_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star + \alpha_{I,n}^j P_{I,n}^\star I_n^\star + \sum_{k=1}^J \gamma_n^{j,k} \left(\sum_{i=1}^N X_{in}^{\star\ k} \right)$	$\forall (n, j$
F4	$P_n^{\star j} I_n^{\star j} = \alpha_{I,n}^j P_{I,n}^\star I_n^\star ;\ P_n^{\star j} C_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star$	$\forall (n, j)$
F5	$IN_n^\star \equiv R_n^\star K_n^\star + W_n^\star E_n^\star N_n^\star + D_n^\star = P_{C,n}^\star C_n^\star + P_{I,n}^\star I_n^\star$	$\forall (n)$
T1	$c_n^{\star j} \equiv \Upsilon_n^j \left[\left(W_n^{\star j} \right)^{\beta_n^j} \left(R_n^{\star j} \right)^{1-\beta_n^j} \right]^{\gamma_n^j} \prod_{k=1}^J P_n^{\star k \gamma_n^{k,j}}$ where $\Upsilon_n^j \equiv \gamma_n^j {\beta_n^j}^{-\gamma_n^j \beta_n^j} \gamma_n^j \left(1 - \beta_n^j\right)^{-\gamma_n^j \left(1-\beta_n^j\right)} \prod_{k=1}^J {\gamma_n^{k,j}}^{-\gamma_n^{k,j}}$	$\forall (n, j)$
T2	$P_n^{\star j} {=} A \cdot \left[\sum_{i=1}^N \lambda_i^{\star j} \left({\kappa_{ni}^\star}^j c_i^{\star j} \right)^{-\theta} \right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma \left(\frac{1+\theta-\sigma}{\theta} \right)^{\frac{1}{(1-\sigma)}}$	$\forall (n, j)$
T3	${\pi_{ni}^\star}^j = \frac{\lambda_i^{\star j} \left(c_i^{\star j} {\kappa_{ni}^\star}^j \right)^{-\theta}}{\sum_{m=1}^N {\lambda_m^\star}^j \left({c_m^\star}^j {\kappa_{nm}^\star}^j \right)^{-\theta}} = \lambda_i^{\star j} \left(\frac{A^j c_i^{\star j} {\kappa_{ni}^\star}^j}{P_n^{\star j}} \right)^{-\theta}$	$\forall (n, i, j)$
T4	$P_{n,C}^\star C_n^\star + P_{n,I}^\star I_n^\star = R_n^\star K_n^\star + W_n^\star E_n^\star N_n^\star + D_n^\star$	$\forall (n)$
T4'	$P_{n,C}^\star C_n^\star + P_{n,I}^\star K_n^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta \right) P_{n,I}^\star K_n^\star + W_n^\star E_n^\star N_n^\star + D_n^\star$	$\forall (n)$
T5	$K_n^\star = I_n^\star + (1 - \delta)\, K_n^\star$	$\forall (n)$
T6	$\sum_{j=1}^J \sum_{i=1}^N {X_{in}^\star}^j - \sum_{j=1}^J \sum_{i=1}^N {X_{ni}^\star}^j = NX_n^\star = -D_n^\star$	$\forall (n, j)$
T7	$D_n^\star = -\phi_n^\star \left(R_n^\star K_n^\star + W_n^\star E_n^\star N_n^\star \right) + \bar{L}_n^\star T^{\star P}$	$\forall (n)$
T8	$\sum_{n=1}^N \phi_n^\star \left(R_n^\star K_n^\star + W_n^\star E_n^\star N_n^\star \right) = \sum_{n=1}^N \bar{L}_n^\star T^{\star P}$	$\forall (n)$

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Panel Regression Results

Under different cohort structure

	Average value in the future 4 years								
VARIABLES	TFP growth rate			Patent.Applications (per 1000 people)			Industrial.Design.Applications (per 1000 people)		
Different age intervals:	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts
3 cohorts:	21.48***	26.22***	25.36***	-1.60***	-1.56***	-1.11***	-0.89***	-0.55***	-0.53***
[0, 14], [15,64], [64,+)	(3.61)	(4.24)	(4.08)	(-4.60)	(-7.06)	(-4.09)	(-3.84)	(-3.87)	(-2.87)
	35.46***	34.48***	31.80***	0.58***	0.18	-1.72***	0.63***	0.71***	0.08
4 cohorts:	(5.19)	(4.28)	(4.35)	(2.73)	(0.46)	(-4.06)	(4.98)	(2.87)	(0.31)
[0,24], [25,49], [50,74],	38.25***	43.60***	34.74***	2.29**	4.90***	3.59***	-0.42	1.08***	1.75***
[75, +)	(3.42)	(4.41)	(3.46)	(2.50)	(7.40)	(6.47)	(-0.98)	(2.93)	(5.20)
		13.47	55.17***		-2.59	4.23***		-1.85**	-0.31
5 cohorts:		(0.90)	(5.35)		(-1.59)	(3.99)		(-1.99)	(-0.46)
[0, 19], [20,39], [40,59],			-21.89			-7.67***			-1.09
[60,79], [80,+)			(-1.08)			(-2.62)			(-0.57)
Initial.Log	-3.46***	-3.51***	-3.51***						
.Dependent	(-4.77)	(-4.49)	(-4.55)						
PoP.Growth									
Observations	732	732	732	395	395	395	215	215	215
R-squared	0.266	0.263	0.272	0.859	0.880	0.886	0.935	0.939	0.942
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Panel Regression Results

Under different cohort structure

	Average value in the future 4 years								
VARIABLES	Cap.Formation(% GDP)			Gross.Consumption(% GDP)			K/L growth rate		
Different age intervals:	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts
3 cohorts:	9.34	16.69***	15.40**	98.55***	92.44***	92.98***	21.77***	24.08***	22.11***
[0, 14], [15,64], [64,+)	(0.98)	(2.64)	(2.32)	(9.21)	(14.84)	(12.62)	(3.69)	(4.50)	(4.02)
	34.10***	29.11***	26.71**	64.81***	60.55***	71.18***	32.98***	39.21***	36.72***
4 cohorts:	(6.74)	(4.14)	(2.52)	(9.15)	(5.58)	(5.39)	(5.32)	(5.36)	(5.30)
[0,24], [25,49], [50,74],	-31.87	37.83**	20.39	98.58***	59.95***	43.58*	8.34	19.18	27.00***
[75, +)	(-1.30)	(2.05)	(1.13)	(2.95)	(3.23)	(1.85)	(0.61)	(1.66)	(2.98)
		-124.60***	53.93**		150.74***	100.97**		4.22	21.25
5 cohorts:		(-2.77)	(2.37)		(3.21)	(2.47)		(0.24)	(1.41)
[0, 19], [20,39], [40,59],			-224.74***			126.47*			-9.87
[60,79], [80,+)			(-3.07)			(1.75)			(-0.33)
Trade Cost							-0.83**	-0.83**	-0.79**
							(-2.13)	(-2.11)	(-2.00)
Initial.Log							-1.99***	-1.98***	-1.93***
.Dependent							(-3.45)	(-3.21)	(-3.14)
PoP.Growth							-33.14*	-35.31**	-30.58
							(-1.84)	(-2.08)	(-1.64)
Observations	724	724	724	725	725	725	758	758	758
R-squared	0.971	0.972	0.972	0.996	0.996	0.996	0.785	0.787	0.787
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Panel Regression Results

Regression Coefficients follows hump shape

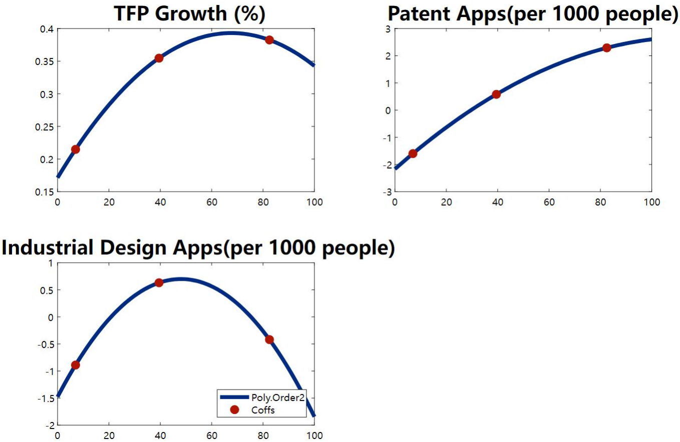

Figure: 3 cohorts: [0, 14], [15,64], [64,+)

Similar hump shape for 4 cohorts and 5 cohorts:

4 cohorts: [0,24], [25,49], [50,74], [75, +) ▸ 4 cohorts

5 cohorts: [0, 19], [20,39], [40,59], [60,79], [80,+) ▸ 5 cohorts

Panel Regression Results

Regression Coefficients follows hump shape

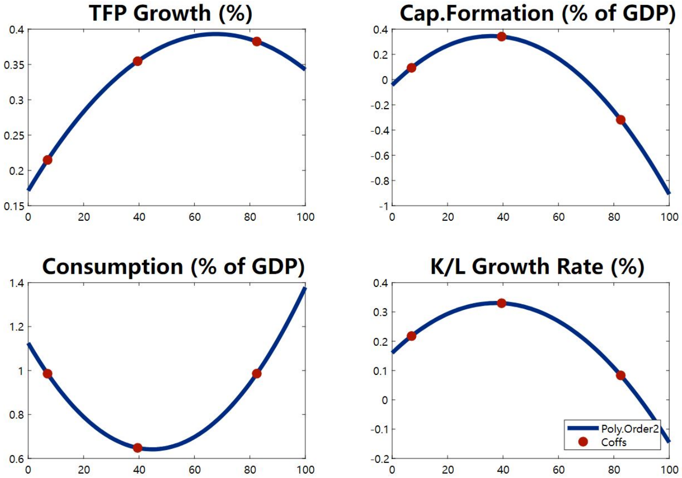

Figure: 3 cohorts: [0, 14], [15,64], [64,+)

Similar hump shape for 4 cohorts and 5 cohorts:

4 cohorts: [0,24], [25,49], [50,74], [75, +) ▸ 4 cohorts

5 cohorts: [0, 19], [20,39], [40,59], [60,79], [80,+) ▸ 5 cohorts

Panel Regression Results

Under different cohort structure

	Average value in the future 4 years											
VARIABLES	TFP growth rate			Cap.Formation(% GDP)			Gross.Consumption(% GDP)			K/L growth rate		
Different age intervals:	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts
3 cohorts:	21.48***	26.22***	25.36***	9.34	16.69***	15.40**	98.55***	92.44***	92.98***	21.77***	24.08***	22.11***
[0, 14], [15,64], [64,+)	(3.61)	(4.24)	(4.08)	(0.98)	(2.64)	(2.32)	(9.21)	(14.84)	(12.62)	(3.69)	(4.50)	(4.02)
	35.46***	34.48***	31.80***	34.10***	29.11***	26.71**	64.81***	60.55***	71.18***	32.98***	39.21***	36.72***
4 cohorts:	(5.19)	(4.28)	(4.35)	(6.74)	(4.14)	(2.52)	(9.15)	(5.58)	(5.39)	(5.32)	(5.36)	(5.30)
[0,24], [25,49], [50,74],	38.25***	43.60***	34.74***	-31.87	37.83**	20.39	98.58***	59.95***	43.58*	8.34	19.18	27.00***
[75, +)	(3.42)	(4.41)	(3.46)	(-1.30)	(2.05)	(1.13)	(2.95)	(3.23)	(1.85)	(0.61)	(1.66)	(2.98)
		13.47	55.17***		-124.60***	53.93**		150.74***	100.97**		4.22	21.25
5 cohorts:		(0.90)	(5.35)		(-2.77)	(2.37)		(3.21)	(2.47)		(0.24)	(1.41)
[0, 19], [20,39], [40,59],			-21.89			-224.74***			126.47*			-9.87
[60,79], [80,+)			(-1.08)			(-3.07)			(1.75)			(-0.33)
Trade Cost										-0.83**	-0.83**	-0.79**
										(-2.13)	(-2.11)	(-2.00)
Initial.Log	-3.46***	-3.51***	-3.51***							-1.99***	-1.98***	-1.93***
.Dependent	(-4.77)	(-4.49)	(-4.55)							(-3.45)	(-3.21)	(-3.14)
PoP.Growth										-33.14*	-35.31**	-30.58
										(-1.84)	(-2.08)	(-1.64)
Observations	732	732	732	724	724	724	725	725	725	758	758	758
R-squared	0.266	0.263	0.272	0.971	0.972	0.972	0.996	0.996	0.996	0.785	0.787	0.787
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

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Panel Regression Results

Coefficients of different cohort

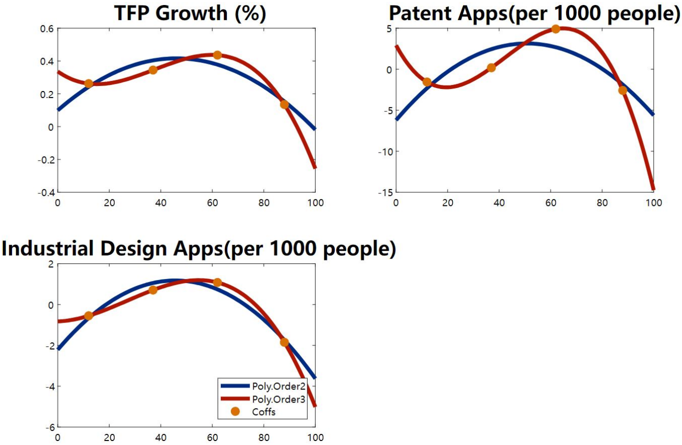

Figure: 4 cohorts

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Panel Regression Results

Coefficients of different cohort

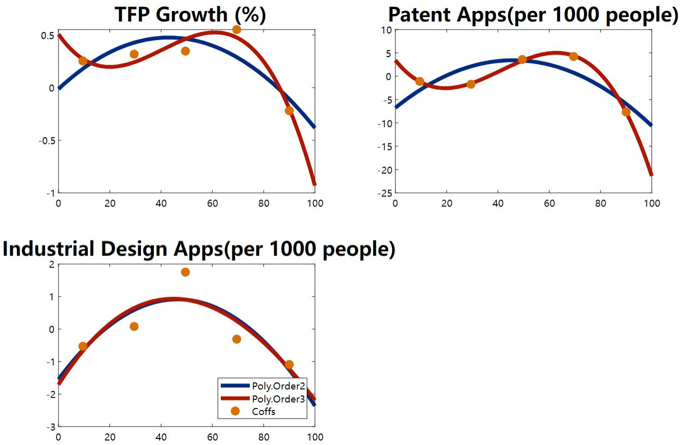

Figure: 5 cohorts

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Panel Regression Results

Coefficients of different cohort

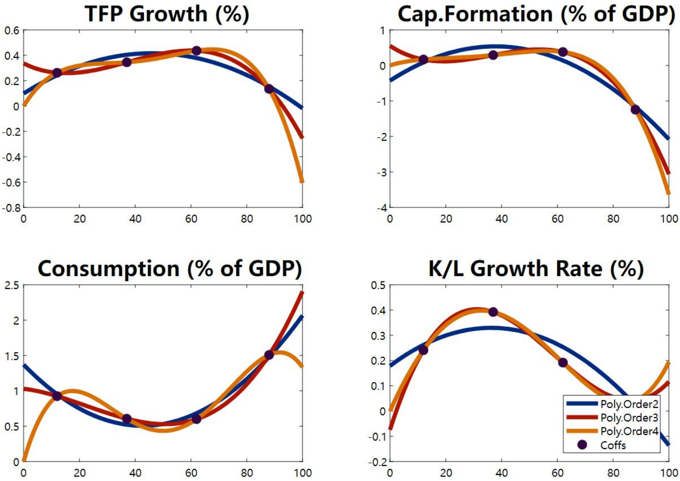

Figure: 4 cohorts

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Panel Regression Results

Coefficients of different cohort

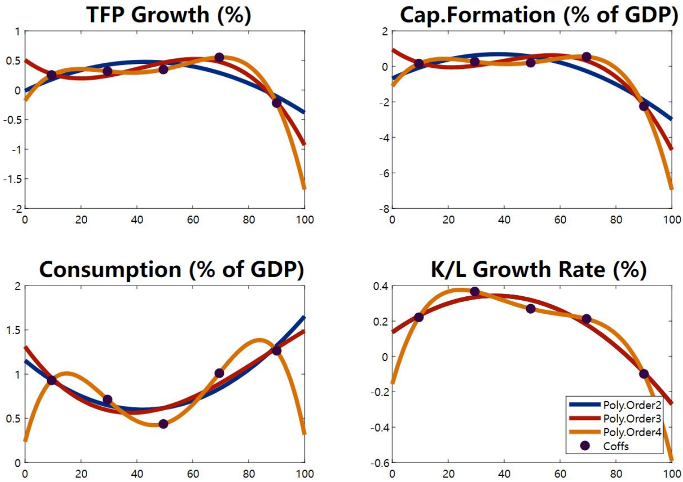

Figure: 5 cohorts

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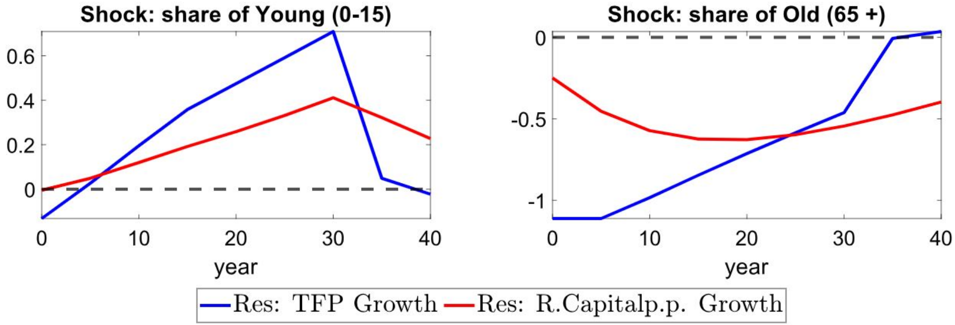
Panel VARX model

IRF of exogenous shock on
I.TFP growth; II. Growth rate of real capital stock per person



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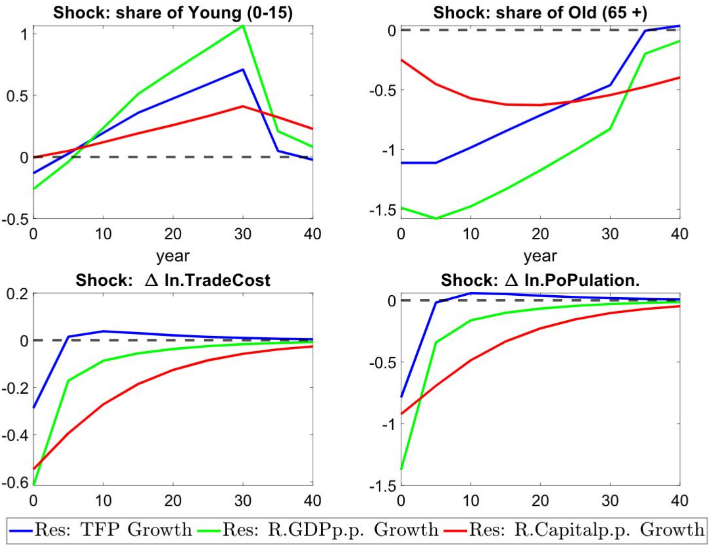
Panel VARX model

IRF of exogenous shock on

I.TFP growth, **II**. Growth rate of Real GDP per person, **III**. Growth rate of real capital stock per person



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Model

Transitional Dynamics

Definition

A competitive equilibrium in the perfect foresight overlapping generations trade model with E+G-period lived agents and exogenous population dynamics, is defined as a series of capital distribution $\{b_{n,g+1,t+1}\}_{g=E+1,n}^{E+G-1}$ and rental rates $R_{t,n}$ and wage rates $W_{t,n}$ satisfies the following conditions:

- The households at different ages taking prices, transfer and deficit as given, optimize lifetime utility
- Firms taking prices as given, minimize production cost
- Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost
- All markets are clear.

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▸ Equations

Transitional Dynamics (1/2)

Table: Dynamic equilibrium conditions (1/2)

H1	$L_{n,t} \equiv \sum_{g=1}^{E+G} \eta_{n,g,t}$; $\bar{L}_{n,t} \equiv \sum_{g=E+1}^{E+G} \eta_{n,g,t}$; $N_{n,t} = \left(1-\tau_{n,t}^{L}\right) \sum_{g=E+1}^{E+G_0} \eta_{n,g,t} l_g = \left(1-\tau_{n,t}^{L}\right) \sum_{g=E+1}^{E+G} \eta_{n,g,t} l_g$	$\forall (n,t)$
H2'	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} b_{n,g+1,t+1} = \left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right) P_{n,I,t} b_{n,g,t} + W_{n,t}\left(1-\tau_{n,t}^{L}\right) E_{n,t} l_g + \frac{TRSV_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}}$; $\forall g \geq E+1$	$\forall (n,t)$
H3	$b_{n,E+1,t} = b_{n,E+G+1,t} = 0$, $c_{n,E+g,t} > 0$, $\{c_{n,g,t+g-1}\}_{g=E+1}^{E+G}$, $\{b_{n,g+1,t+g}\}_{g=E+1}^{E+G-1}$	$\forall (n,t)$
H4	$TRS_{n,t}^{V} = \frac{R_{n,t}}{P_{n,I,t}} \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1,t-1} - \eta_{n,g,t}\right) b_{n,g,t}$; $TRS_{n,t}^{D} = (1-\delta) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1,t-1} - \eta_{n,g,t}\right) b_{n,g,t}$	$\forall (n,t)$
H5	$TRSV_{n,t} = P_{n,I,t}\left(TRS_{n,t}^{V} + TRS_{n,t}^{D}\right) = P_{n,I,t}\left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1,t-1} - \eta_{n,g,t}\right) b_{n,g,t}$	$\forall (n,t)$
H6	$\left(\frac{c_{n,g+1,t+g}}{c_{n,g,t+g-1}}\right)^{1/\sigma} = \beta\left(\frac{\psi_{n,t+g}}{\psi_{n,t+g-1}}\right)\left(1+\frac{R_{n,t+g}}{P_{n,I,t+g}}-\delta\right) \frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}}$ for $\forall\ g \in [E+1, E+G-1]$	$\forall (n,t)$
H7	$C_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} c_{n,g,t}$; $I_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} i_{n,g,t}$; $K_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g-1,t-1} b_{n,g,t}$	$\forall (n,t)$
F1	$W_{n,t} E_{n,t} N_{n,t} = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^{j} X_{i,t}^{j}$	$\forall (n,t)$
F2	$R_{n,t} K_{n,t} = \sum_{j=1}^{J} \left(1-\beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^{j} X_{i,t}^{j}$	$\forall (n,t)$

Model

Transitional Dynamics (2/2)

Table: Dynamic equilibrium conditions (2/2)

F3	$X^j_{n,t} = \alpha^j_{C,n} P_{C,n,t} C_{n,t} + \alpha^j_{I,n} P_{I,n,t} I_{n,t} + \sum_{k=1}^J \gamma_n^{j,k} \left(\sum_{i=1}^N X^k_{in,t} \right)$	$\forall (n,j,t)$
F4	$P^j_{n,t} I^j_n = \alpha^j_{I,n} P_{I,n,t} I_{n,t}$; $P^j_{n,t} C^j_n = \alpha^j_{C,n} P_{C,n,t} C_{n,t}$	$\forall (n,j,t)$
F5	$IN_{n,t} \equiv R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} + D_{n,t} = P_{C,n,t} C_{n,t} + P_{I,n,t} I_{n,t}$	$\forall (n,t)$
T1	$c^j_{n,t} \equiv \Upsilon^j_n \left[\left(W^j_{n,t} \right)^{\beta^j_n} \left(R^j_{n,t} \right)^{1-\beta^j_n} \right]^{\gamma^j_n} \prod_{k=1}^J P^k_{n,t}{}^{\gamma_n^{k,j}}$ where $\Upsilon^j_n \equiv \gamma^j_n \beta^j_n{}^{-\gamma^j_n \beta^j_n} \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n (1-\beta^j_n)} \prod_{k=1}^J \gamma_n^{k,j}{}^{-\gamma_n^{k,j}}$	$\forall (n,j,t)$
T2	$P^j_{n,t} = A \cdot \left[\sum_{i=1}^N \lambda^j_{i,t} \left(\kappa^j_{ni,t} c^j_{i,t} \right)^{-\theta} \right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma \left(\frac{1+\theta-\sigma}{\theta} \right)^{\frac{1}{(1-\sigma)}}$	$\forall (n,j,t)$
T3	$\pi^j_{ni,t} = \frac{\lambda^j_{i,t} \left(c^j_{i,t} \kappa^j_{ni,t} \right)^{-\theta}}{\sum_{m=1}^N \lambda^j_{m,t} \left(c^j_{m,t} \kappa^j_{nm,t} \right)^{-\theta}} = \lambda^j_{i,t} \left(\frac{A^j c^j_{i,t} \kappa^j_{ni,t}}{P^j_{n,t}} \right)^{-\theta}$	$\forall (n,i,j,t)$
T4	$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T4'	$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta \right) P_{n,I,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T5	$K_{n,t+1} = I_{n,t} + (1-\delta) K_{n,t}$	$\forall (n,t)$
T6	$\sum_{j=1}^J \sum_{i=1}^N X^j_{in,t} - \sum_{j=1}^J \sum_{i=1}^N X^j_{ni,t} = NX_{n,t} = -D_{n,t}$	$\forall (n,j,t)$
T7	$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) + \bar{L}_{n,t} T^P_t$	$\forall (n,t)$
T8	$\sum_{n=1}^N \phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) = \sum_{n=1}^N \bar{L}_{n,t} T^P_t$	$\forall (n,t)$

Equations